Understanding Electricity and Circuits: What the Text Books Don't Tell You

Ian M. Sefton

School of Physics, The University of Sydney

I.Sefton@physics.usyd.edu.au

Part 1 Introduction

Most of the standard physics text books that we all know and either love or hate have some serious deficiencies. My particular beef here is that, by trying to oversimplify some basic physics, those books introduce or encourage some serious misconceptions and tell stories that are hard to believe. For this discussion I have chosen the topic of simple circuits as exemplified by a battery and a small torch globe – can we find a simpler circuit than that? I will use that example to explore some really important physics that all school-level and most junior university-level texts omit and to confront a serious misconception that can arise from studying those texts.

There are four main parts to this article. In part 1 I introduce the example and in part 2 we have a look at the misconception about energy transfer together with a quick summary of a better model. In parts 3 and 4 we will examine the basic physics in more detail and justify the alternative model by applying the principles to the example in more detail. For an overview of the problem and its resolution you need only read parts 1 and 2.

The topic for discussion is the simple circuit shown in figure 1. It consists of a dry cell (which everybody now calls a battery), a switch, a torch globe and some wires. We want to study how energy gets from the battery to the globe.

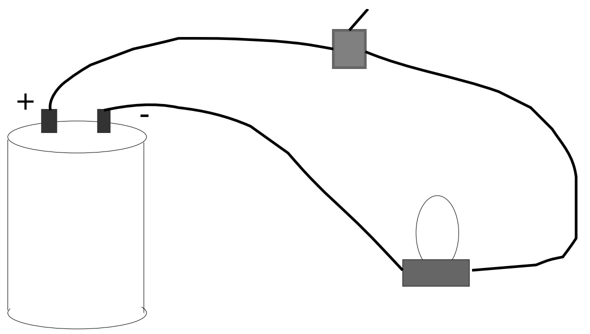

Figure 1. Battery and globe circuit

How does energy get from the battery into the globe?

Part 2 Demolition of a Myth

2.1 The basic misconception

The core misconception, propagated by many text books, is that moving electrons (or some other form of current) in the connecting wires carry energy from the battery to the globe. I can give at least five arguments why that idea is conceptually unsound. You may be able to think of more.

Objection 1 is that that electrons are just too slow to carry the energy fast enough! When the switch is closed the light globe comes on almost at once. Many text books discuss a model of electrical conduction in which a "gas" or "sea" or electrons is pushed slowly along a wire by an electric field. If you know the density of electrons (the number of conduction electrons per volume of wire), the diameter of the wire and a typical current you can work out how fast the electron sea moves along. In a typical example of a 1mm copper wire carrying a current of 100 mA the answer turns out to be about 0.01 $mm.s^{-1}$ which is much slower than a tortoise. If those electrons were picking up energy from the battery and then carrying it all the way to the light globe, you would have to wait an awfully long time to see the globe light up.

Objection 2 looks at AC circuits in which electrons don't go anywhere much; they just jiggle back and forth. So they can't carry energy from one place to another. It would be silly to have a basically different theory for AC and DC.

Objection 3: although some books say that you have to have a complete conducting loop before a current can exist, that is just another misconception. Electrons do not travel across the insulating gap in a capacitor nor do they jump across the space between the primary and secondary windings of a transformer. This is so even when the energy source is a battery; I have constructed circuits like those in figure 2 that show that the lamp lights up briefly when the switch is closed. No matter how the energy travels in those examples, it must be able to get through empty space. (It is true that if you want to maintain a steady current in a circuit, then a continuous conducting loop is required.)

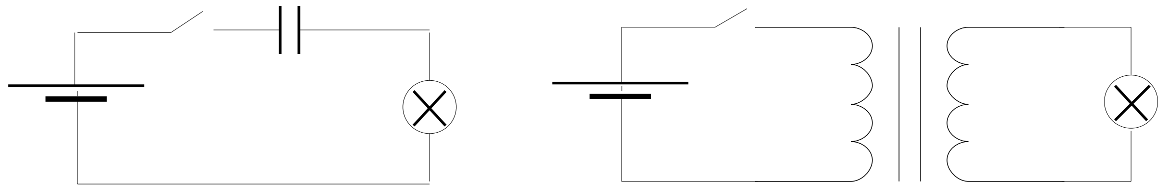

Figure 2. Two circuits with a non-conducting gap

With suitable component values you can make the globe light up when the switch is closed, but there is no path for electrons to travel from battery to globe.

Objection 4 says that energy transfer is a one-way process, but current goes round a loop. If electrons in one wire carry energy from the battery to the globe, why don't the electrons in the other wire carry energy from the globe back to the battery? There is no way of associating the direction of energy flow with the direction of the current or with the direction in which the electrons drift.

Objection 5 points to another basic misconception that is often implied and sometimes explicitly stated in texts. The mistake is to speak of the electric potential energy (PE) of an electron as though the electron owns all the PE – it doesn't. This error seems to be a reflection of a similar sloppy way of talking about gravitational PE. When you lift a brick do you increase its PE? No, you increase the PE of the system that consists of the two objects: Earth and brick. PE is not stored in either Earth or the brick nor is it possible to apportion the PE between the two interacting objects. The first step to enlightenment is to learn not to speak or think about gravitational or electrical PE of electrons (or of any other kind of particle) and think instead of energy as a property of the whole system. If you really insist on knowing *where* the energy resides I will come back to that issue later.

To summarise the objections: electrons don't have PE of their own, they travel very slowly in both directions between battery and globe, and in a circuit they generally don't go anywhere much but energy is transferred very rapidly and it can get across gaps in the conducting path. The same goes for forms of current other than electron drift.

2.2 The nature of energy

The question about what carries the energy seems to be linked to a common conception that energy is like matter: put it in one end of a pipe and watch it come out the other end. At a truly basic level energy is not like that; it is just an abstract attribute of a system that, when calculated in appropriate ways, always turns out to be conserved.

There are two valid schools of thought about how one should conceptualise energy and energy transfer, that I will call the "accountant's model" and the "field model".

The accountants view energy solely as a mathematical attribute of physical systems, and it does not need any kind of conceptual model beyond that. They will tell you that it is a mistake to think of energy as a kind of substance. It is just an abstract quantity that, when you calculate it properly, always tallies. In this conceptual model energy is nothing like matter, it is just a mathematical abstraction that expresses some aspects of the behaviour of the natural world. In the case of our circuit the accountants will answer: "Who says that anything has to carry the energy? That's the wrong question, because all we have to do is account for changes in energy in a kind of balance sheet – the precise location of the energy is not important and may be unknowable." The accountant's attitude is in the same tradition of thinking as the idea of action at a distance in which, for example, we don't seek to understand how Earth's gravity can reach out and grab the Moon (and vice versa). We just accept as a fact that it does. Similarly we don't ask where the potential energy of the Earth-Moon system might be; it just belongs to the system. Taking the same conceptual view, it is grossly incorrect to talk of the PE of an electron.

If we apply this whole-system, action-at-a distance, model to our simple circuit the question of precisely how energy gets out of the battery and into the globe is not answerable. End of story.

2.3 Energy and fields

Such an abstract view of energy as the accountant's may not be satisfying to students who seek more concrete conceptions. Fortunately for them there is another kind of conceptual model that allows us to say that energy actually leaves the battery and ends up in the globe and we should be able to track where it goes. This idea that you can actually track energy flow across boundaries to make sure that the total value is conserved is called **local conservation of energy**, a much stronger law than just saying that energy is conserved in a way that need not concern us.

The "field view" of energy is more in the realist tradition of conceptual thinking about physics. It is still incorrect to conceive of energy as a substance-like entity that occupies space, but there are entities called fields that do occupy space. The fields of interest in our case are the electric and magnetic fields described by the "classical" electromagnetic theory of Clerk Maxwell and others, way back in the 19th Century. These fields are most unlike any kind of substance – they are vector quantities. According to Maxwellian theory, electric and magnetic fields are closely related to each other and both are associated with energy. The density of energy in space, energy per volume, can actually be calculated if you know the details of each field. (The same thinking can be applied to gravity – PE is stored in the gravitational field.) Furthermore, Maxwell's theory also predicted the existence of electromagnetic waves travelling at the speed of light, which led to the idea that light itself may consist of electric and magnetic fields. In this way of modelling nature it is legitimate to say that light and other electromagnetic waves carry energy.

2.4 The field explanation of energy transfer

The idea that the electromagnetic field both stores and transmits energy is the key to the explanation of how energy gets from the battery to the light globe. Before going into detail of the theory, let's look at the explanation in a nutshell. The narrative involves four conceptual characters: *charge, current*, *electric field* and *magnetic field* (italicised in the summary below) whose complex relationships are governed by four equations (known as Maxwell's equations). These four characters are really played by just two actors, each of whom has two faces. *Current* is just *charge* in motion while *electric field* and *magnetic field* are but two aspects of a fundamental entity which can be called electromagnetic field. Except for static scenes in which nothing much ever happens (called electrostatics) any good story about electromagnetic phenomena almost inevitably involves all four characters.

The story of energy transfer from the battery to the globe goes like this. When the battery is first connected to complete the circuit it pushes electrons (*charge*) around so that they pile up on the surfaces of some parts of the circuit, leaving a deficit of electrons, and hence a positive *charge* on other parts of the conductors' surfaces. This pushing around of electrons is mediated by the *electric field.* The *charge* separation in turn produces *electric field* inside the connecting wires as well as in the wire filament of the light globe. The internal *electric field* is directed along the axis of the wires and is responsible for producing a drift of mobile charge carriers, *current*, in the wires. To explain energy transfer we need to look at what is happening outside the wires. As a consequence of the surface charges on the wires, there is an *electric field* in the space outside the wires (as well as inside). Also, as a consequence of having a *current* in the wires, there is a *magnetic field* in the space around the wires. It is this combination of *electric field* and *magnetic field* in the space outside the wires that carries the energy from battery to globe. Once the fields are set up, the energy travels through space, perpendicular to both the *electric field* and the *magnetic field*, at the speed of light. Energy leaves through the sides of the battery and enters the wire of the globe through the sides of the wire.

As Arnold Sommerfeld (1952) has pointed out, metals are good conductors of current but non-conductors of energy. Metals conduct current but space conducts energy and the best conductor of electromagnetic energy is the vacuum!

This story answers all the objections listed above. There is no need to wait for electrons to complete a journey (objections 1 and 2) because the electrons are not given the energy in the first place (answer to objection 5). It is a normal part of the story that electromagnetic energy can cross space (including the space in transformers and capacitors), which needs to contain nothing but electric and magnetic fields (answer to objection 3). It also turns out that, when we map out the paths along which the energy flows through space, it is a one-way traffic in the correct direction (answer to objection 4).

The explanations of what causes current and of how energy gets transferred are intimately linked. The rest of this article is devoted to an elaboration and justification of the two stories, starting with the explanation of current.

Part 3. Modelling the Circuit

Since the filament of an incandescent globe is made of wire, the story of what happens in our battery-and-globe circuit can be modelled by considering what happens in a piece of wire that carries a current. Whatever applies to a straight piece of wire can be adapted to the complicated geometry of a real circuit by combining many short pieces of wire.

3.1 Modelling current

Current is usually defined as flow of charge. However, since charge is not something that exists on its own but is a property of objects, current is more properly defined as a flow of charged particles. Although current can exist as a stream of charged particles in otherwise empty space, we are concerned here with currents within material conductors such as copper and tungsten wires. The common model of such conductors has a lattice of positively charged metal ions that are more or less fixed within the conductor, permeated by a "sea" of mobile, negatively charged, electrons. Normally the total charge on the whole conductor is zero. Current in a metal consists of the slow drift of the electron sea through the fixed lattice. Electrical interactions between ions of the lattice and conduction electrons hinder the flow of electrons, giving rise to the property of electrical resistance. To maintain a current, therefore, there must be something pushing the electrons along against the resistance of the lattice.

3.2 Charge separation, electric field and potential difference

In circuits driven by batteries the push on the conduction electrons comes ultimately from a separation of charge produced and maintained by the battery, but the best way to describe the push at the level of individual electrons is to say that there must be an electric field inside the wire. This field produces a force on each conduction electron. (Recall the definition of electric field using $\boldsymbol{F}=q\boldsymbol{E}$.)

The next question is: what produces the electric field inside the wire? There are only two ways of creating an electric field. One way is to have a changing magnetic field, as in the phenomenon of electromagnetic induction. That mechanism does not operate in our example, so the electric field must be caused by the other mechanism, separation of positive and negative charge. That is just what batteries do.

It is instructive to look at the static situation in our example (figure 1), before the switch is closed. By internal processes of diffusion and chemical reactions, which we will not go into here, a battery produces and maintains a separation of electric charge between its terminals. We have a piece of wire connected to the battery's positive terminal with another wire and the globe connected to its negative terminal. Those wires and the globe will each acquire a net charge that sits on their surfaces. Although it is not easy to calculate exactly how those charges are spread around, we do know that the total charge on the system is zero. The positive battery terminal and its wire have a deficit of electrons that is exactly matched by a surplus on the negative terminal, the negative wire and the globe. Charge has been separated by the action of the battery and that charge separation is responsible for the existence of an electric field in the space around the system. The electric field also exists inside the battery where it acts to oppose the processes that produce the charge separation. However, if there is no complete circuit, we have a purely electrostatic situation in which the surface charges arrange themselves so that like charges try to get far away from each other and there is no electric field inside the wires or the globe.

The special thing about a battery is that when a circuit is completed there is a conducting path through which the separated charges can get back together, but the battery continues to push opposite charges apart, and the situation is no longer purely electrostatic. The distribution of charge on the surfaces of the conductors changes in such a way that electric field now exists inside the wires and the globe's filament (as well as in the space outside). That electric field drives the current.

The bit of the story that most text books omit is that the surfaces of all the conductors, in our the case the copper wires and the globe's filament, must all have some distribution of charge on their surfaces. (A notable exception to this criticism is the text by Chabay & Sherwood, 2002).

Standard texts often tell us that a potential difference (PD) drives the current, which is true because potential difference is simply related to the electric field. In a wire where the electric field has magnitude E, the PD, V, between two points separated by a distance d along the wire is given by: $V = Ed$. (Remember though, that in describing what happens in a wire, electric field is more basic than PD, because it is also possible to get electric field from electromagnetic induction; PD exists only when there is charge separation.) It is important to remember that potential and electric field both exist in space, and the field has a unique value (which consists of a magnitude and a direction) at every point.

3.3 Surface charges on conductors

It is worth having a closer look at the kind of arrangement of surface charge that can produce electric field inside wires, because those charge distributions also affect the electric field in the space outside the circuit. If the current is to stay inside the wire, as it must, the field has to point along the axis of the conductor in the same direction as conventional current (the flow of equivalent positive charge). A distribution of surface charge whose concentration varies linearly with distance along the wire will do the trick nicely (see figure 3 and its caption).

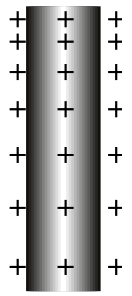

Figure 3. One possible distribution of surface charge on a straight wire

A free electron at the bottom of this wire will be pulled towards the higher concentration of positive surface charge at the top. Distributions with negative charge are also possible.

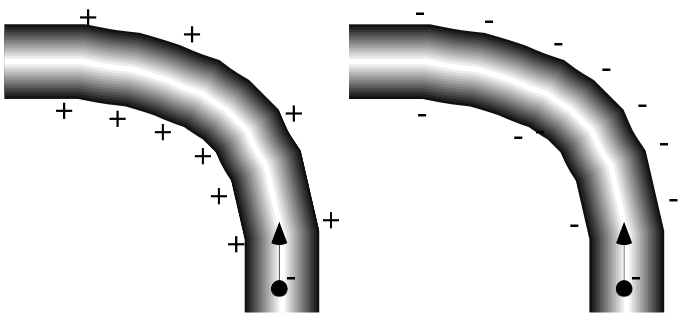

Figure 4. Two ways to push a conduction electron around a corner

The surface charges shown are in addition to those which produce a field directed along the axis of the straight parts of the wire.

At bends in the wire there must be some additional unbalanced charge in order to change the direction of the field and the current. To see how a distribution of surface charge can direct the flow of mobile charges and also adjust itself to achieve that result, imagine an electron drifting a long a wire and approaching a bend. The only way that it can get around the bend on a smooth path is for it to be pushed or pulled around the corner by some other charged particles that are lying in wait for it. If that were not so, the electron would keep going and run into the surface of the wire. If it does that and sticks there it may, in turn, lie in ambush for another electron coming along behind it. So the first electron, which does not quite make it around the bend, will help to push the next one around. This process may continue until the surface charge builds up just enough to keep all the conduction electrons on track. In ways like this the whole system will very quickly adjust itself until there is just the right distribution of surface charge to ensure a smooth and continuous drift of conduction charge along the wire. The reason that everything adjusts so quickly is that changes in electric field propagate at the speed of light.

The surface charges, which are required to maintain the current, also produce electric field in the space outside the wires and we will see below that this external electric field plays a key role in the transfer of energy.

A detailed mathematical analysis gives the remarkable result that inside a uniform wire the electric field has the same magnitude everywhere inside the wire and has a direction that follows the wire's long axis, i.e. it stays parallel to the axis. This kind of argument shows that the detailed distribution of surface charge depends not only on the composition of all the circuit elements but also on their geometrical arrangement. On the other hand, things that occur inside wires, uniform field, uniform spread of current across a wire, voltage drop proportional to resistance – all the familiar circuit results – seem to be so simple. Perhaps that is the excuse that text writers use for not telling you about surface charge.

Part 4. Electromagnetic Fields and Energy

There are two key ideas connecting the electromagnetic field with energy. First is the idea that electric and magnetic fields store energy and the second is that the two kinds of field together are responsible for the transmission of energy. Both of these ideas originated in the nineteenth century but some text books have not caught up yet.

4.1 Energy storage in fields

In the field model of energy, as in the accountant’s view, it is still misleading to think of energy as a kind of substance. If you want to know details of where the energy is or where it might be going you first need to know the details of both the electric and magnetic fields. Those fields are both vector quantities which each have a value at every point in space and obey the principle of superposition. That principle says that to find the total field due to different sources you just perform a vector addition of all the individual contributions. For example to find the total electric field due to two charged particles, you first calculate the field that would exist at each point in space for each of the particles separately; then you add the contributions at each point. Only after you have calculated the resultant electric field can you find the energy associated with that total field. There is a similar rule for magnetic fields. Two very simple-looking formulas give the energy density (energy per volume, U_E) of the electric field, $\boldsymbol{E}$, and the energy density, U_B, of the magnetic field $\boldsymbol{B}$, at each point in space:

$$U_E = \tfrac{1}{2}\varepsilon_0 E^2,$$

$$U_B = \frac{1}{2\mu_0} B^2.$$

(The constants ε_0 and μ_0 are the same ones that appear in Coulomb’s law and the rules connecting magnetic fields with currents.) If energy were conserved in the same way as a substance like water you should be able to add the individual energy distributions – but that does not work. These formulas tell me that field is more fundamental than energy. They also provide an answer to the problem of locating potential energy. If you have a system of interacting charged particles and if the configuration of the system changes, the particles will do work on each other and the potential energy of the system will change. That change in PE can be described exactly by changes in the energy associated with the fields as described by the two formulas above.

But that’s not the whole story; energy may also leave a system and we can also describe that in terms of the fields.

4.2 Energy transfer by the electromagnetic field

The theory in this section is the ultimate description of energy transfer. Electromagnetic theory predicts that there will be a flow of energy through any place where electric and magnetic fields both exist and are not parallel to one another. A way to picture this energy flow is to imagine a tiny surface, drawn at the chosen point in space (figure 5), which contains both the electric field vector and the magnetic field vector (recall that any two lines define a plane). The energy flow is perpendicular to that surface. Given that there are two directions normal to the surface, we will need to determine the correct one by looking at the relative orientation of the electric and magnetic fields.

If the space is empty or filled with non-magnetic material, a simple-looking vector formula sums it up: the energy flux density (energy per area per time) is described by a new vector quantity:

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B}.$$

The vector product $E\times\boldsymbol{B}$ in this formula is defined to be another vector which is perpendicular to both $\boldsymbol{E}$ and $\boldsymbol{B}$ (i.e. perpendicular to our imaginary surface) and has magnitude $EB\sin(\phi)$, where ϕ is the angle between the field vectors $\boldsymbol{E}$ and $\boldsymbol{B}$. See figure 5. To find which of the two possible directions is appropriate for the product we use a right-hand rule: curl the fingers of the right hand from the direction of the first vector to the second, in this case from $\boldsymbol{E}$ to $\boldsymbol{B}$; the thumb then shows the direction of $\boldsymbol{S}$.

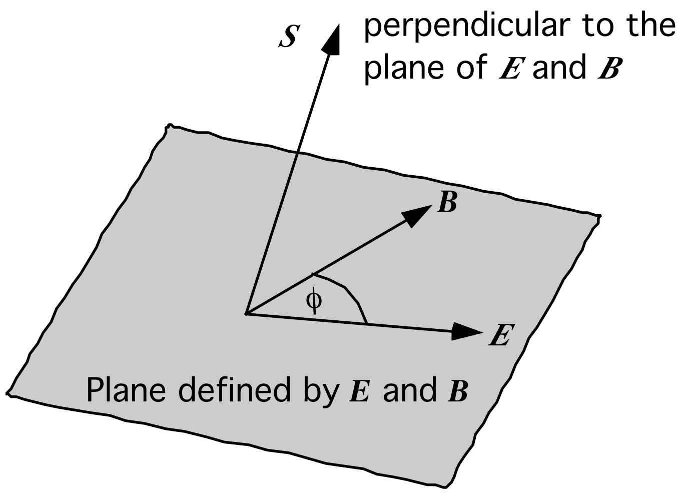

Figure 5. Definition of the energy flux density (Poynting vector)

This vector represents power per area transmitted by the electromagnetic field.

The quantity on the right-hand side of this equation is also known as the Poynting vector in honour of J. H. Poynting (1852-1914) who developed this bit of theory.

In the field model of energy we can still say that PE is an attribute of a system, but we now view the system as consisting of not only the interacting bodies or particles, but also the fields that mediate the interactions. It is also feasible to locate the PE and satisfy local conservation of energy. We now have a way of describing the transfer of electromagnetic energy between systems or parts of a system.

It is worth noting that this description of energy transfer applies to electromagnetic waves, including light. In the classical description of such a wave the electric and magnetic fields are perpendicular to each other and both are also perpendicular to the wave's direction of propagation. It is in that context that the Poynting vector is usually introduced in the more advanced texts, which generates the unfortunate misconception that it applies only to waves.

As we shall see below, the Poynting vector provides the key to understanding energy transfer in DC circuits. But first we need to examine some details of the electric and magnetic fields.

4.3 The external electric field

To study the electric field near the surface of a conducting wire it is helpful to think in terms of components of the field. We can think of two vector components, one parallel to the axis of the wire and one perpendicular to the wire.

You may also recall from a study of electrostatics that if you charge up a conducting body and just leave it alone, it rapidly settles into an equilibrium in which all the excess charge sits on the surface and there is no electric field inside the conductor (and no current either). But there is a field outside and that field is perpendicular to the surface. This electrostatic situation for a section of long straight wire is shown on the left of figure 6.

I have already argued that, inside a wire where there is a current, the electric field must be parallel to the axis of the wire. According to electromagnetic theory, that parallel (axial) component of the field must also exist just outside the wire. Furthermore, the axial component must have the same value just inside and just outside the surface.

When there is a charge distribution on the surface and a current in the wire, both components of the field exist. Inside the wire there is only the axial component, but outside there is a perpendicular (radial) component as well. (To see that the radial component must still be zero inside the wire, think how it would push mobile charge carriers off their required track if it were not.) See the middle sketch in figure 6.

The net result is that although the field inside the conductor is parallel to the current, outside the charged surface of the wire the total electric field may be at an angle to surface (as shown in the right-hand diagram of figure 6).

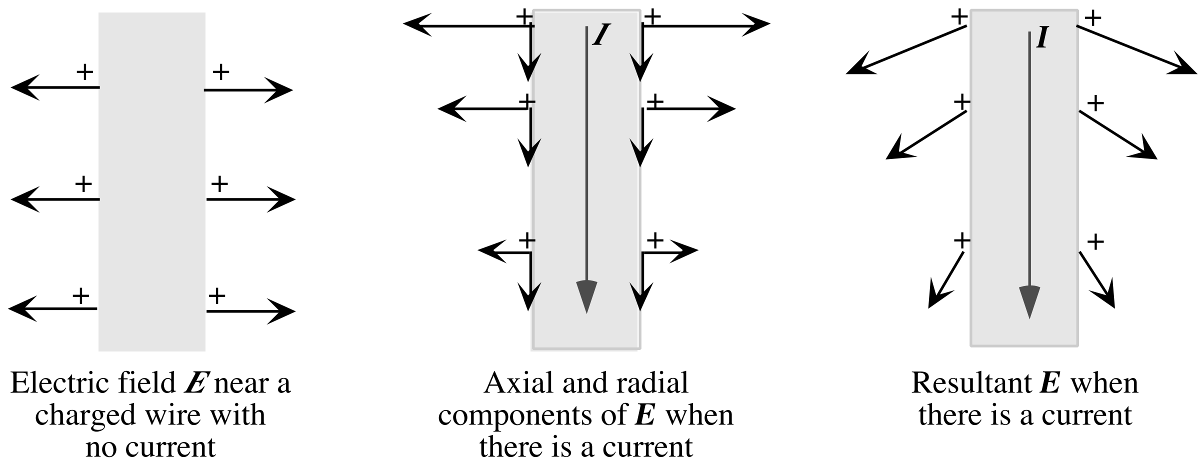

Figure 6. Electric field at the surface of a wire

This piece of wire has resistance. When there is no current, the electric field must be zero inside. A current requires a non-uniform spread of charge on the surface. The sketch in the middle shows, separately, the axial component of electric field at the surface and the perpendicular (radial) component. The sketch at right shows the resultant of the two components.

It is worth noting what the field is like in a wire with very low resistance. In that case, the axial electric field needed to push the charge carriers is also very small, so the axial component at the surface is also very small. However the wire may still be charged, so the total electric field outside the wire must be nearly perpendicular to the surface. We expect therefore that, near the copper connecting wires in our simple circuit, the electric field will point almost radially, either inwards or outwards.

4.4 Energy transfer between battery and globe

The actual distributions of the electric and magnetic fields can be quite difficult to calculate, especially since the details depend on the geometry of the circuit. However it is still possible to get an idea of what is going on by making some rough sketches. Figure 7 shows a simplified representation of the battery and bulb circuit squashed into a plane and forced into a rectangular format – just like those abstract circuit diagrams that we usually draw. We can guess roughly how the electric field lines should go, from the positively charged wire down to the negative wire. The magnetic field lines produced by the current form circles around the wires and point into the page at all points inside the loop. Working out the direction of the Poynting vector according to the rule above we find that inside the loop of the circuit the energy flux density vector always points in the general direction away from the battery towards the globe.

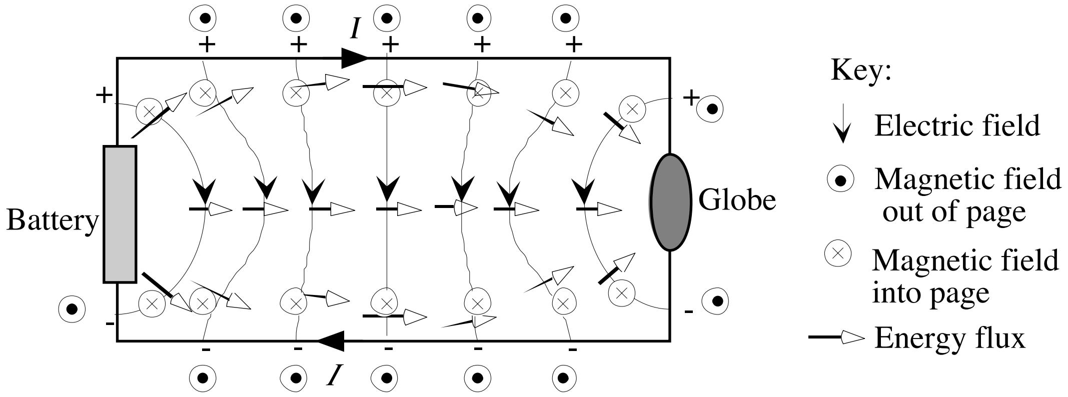

Figure 7. Sketch of field directions and energy flux for the plane region enclosed by the circuit

Fields outside the enclosed area and the strong electric field at the filament of the globe have been omitted for simplicity.

There is also some energy flux in the space outside the loop and, indeed, in the space above and below the flattened circuit of figure 7. If you work out the direction of the Poynting vector in those places you will still see energy leaving the battery and eventually heading in towards the globe. It is interesting to note that the energy takes many different paths, but it is fairly easy to see that the flux density has its largest values just outside the wires where the magnetic field is greatest. The current seems to act like a guide for the energy transfer, but does not allow it get too close

Here is an interesting fact. You may know that every point in space can be assigned a unique value of electric potential and that one can mentally construct imaginary surfaces which each have the same value of the potential. The electric field associated with the potential is always perpendicular to the equipotential surfaces. Since the Poynting vector is also perpendicular to the electric field, we can see that the energy flux follows equipotential surfaces.

4.5 Energy transfer into a piece of wire

A nice conceptual model is of no use unless it also stacks up quantitatively with experiments and other valid theories. So here is an illustration of how the field model of energy transfer gives correct answers. As with many formal derivations in physics, it helps to find a somewhat idealised, but still realistic, example.

To have a look at how energy gets into a light globe we consider the related case of a segment of wire (figure 8) that is nice and straight instead of being coiled up tightly. We need to look at the electric and magnetic fields **just inside** the surface of the wire. As we have already seen, the electric field ***E*** inside the wire points along the axis of the wire and, just inside the surface, the lines of ***E*** lie along the surface and parallel to it (straight grey lines in the diagram). The lines of the magnetic field ***B*** form closed loops around the wire. The diagram shows some of these lines just inside the surface of the wire. Now the energy density vector (***S***) must be perpendicular to the lines of both ***E*** and ***B*** so it must be everywhere perpendicular to the wire's surface.

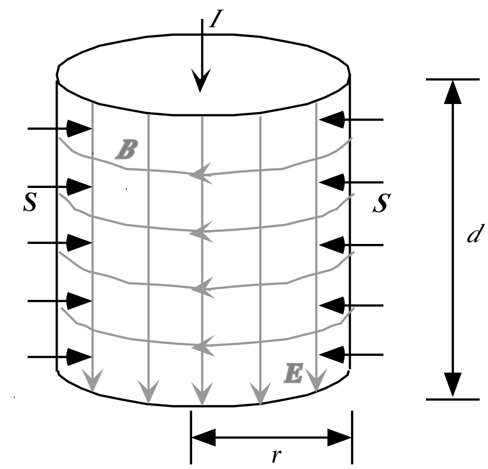

Figure 8. Energy flux density and fields just inside the surface of a wire

Only the axial component of the electric field is shown here because that is the part responsible, together with the magnetic field, for energy flux into the wire.

The diagram shows a few arrows to represent ***S*** which is perpendicular to the surface at every point**.** You can also check whether it should be pointing in or out. The directions of the electric and magnetic fields are both related to the direction of the current. Using the right-hand rule for the vector product stated above, we can conclude that energy is flowing into the wire at every point on its curved surface.

To calculate the value of the power transfer, just multiply the value of the Poynting vector by the surface area. Since the electric and magnetic fields are right angles to each other, the magnitude of the energy flux density is just

$$S = \frac{1}{\mu_0} EB \quad .$$

The power input is the product of the power per area, *S*, and the curved surface area of the cylinder, *A*:

$$P = SA = \frac{1}{\mu_0} EB.2\pi rd \ .$$

Now we can find values for the fields in terms of the PD and current. The magnitude of the electric field is the PD between the ends of the segment of wire divided by the length of the wire,

$$E = \frac{V}{d}$$

and the magnetic field produced by the current at the surface of the wire is given by the well-known formula

$$B = \frac{\mu_0 I}{2\pi r}.$$

Substituting these two expressions for the fields gives the familiar formula for the power dissipation:

$$P = VI \, .$$

So the field model for energy transfer gives the right answer for energy dissipation in a resistive wire. Does this convince you that the field model for energy transfer is a good one?

As an extension to the derivation above, you could look at what happens just outside the wire. Since the axial component of the electric field is the same just outside as it is inside and the magnetic field is practically the same, we can conclude that the energy flux derived above still works out as expected. But ... now there is also a radial component of the electric field as well. If that radial component is not zero then, together with the magnetic field, it contributes to a component of the Poynting vector that is parallel to the wire – but only outside the wire. That means that energy may also be travelling along beside the wire (and getting in somewhere else).

4.6 Energy transfer out of the battery

A similar mathematical argument can be used to find the rate at which energy is extracted from the battery by the electromagnetic field. Just model the battery as a cylindrical object and the whole argument is the same except for one thing: in the battery the current goes from low potential to high potential, against the electric field, so the direction in which the magnetic field loops around the battery is reversed. That's the only difference in the argument, but it reverses the direction of the Poynting vector at the surface. So energy comes out of the cylinder instead of going in. Conservation of energy rules!

4.7 Energy flux between battery and globe

The details of the energy flow in the space between battery and globe depend on the geometry of the circuit, which determines details of how the surface charge gets distributed around the circuit. The remarkable thing is that the simple relations among voltage, current, resistance and power transfer are unchanged by any geometrical rearrangement of the wiring. It is possible to work out the details of some simple cases analytically.

One case where it easy to see where the energy goes is that of a coaxial cable such as that used to connect TV aerials. Imagine that the battery is connected to the globe using a piece of coax and look at the pattern of electric and magnetic fields as shown in cross-section in figure 9. The inner wire is connected to the positive terminal of the battery. Current comes up the central wire and goes back through the outer sheath.

The total magnetic field is zero everywhere in the space outside the cable (the contributions from the two conductors cancel exactly). The surface charge on the outer conductor sits on its inner surface so that the electric field cannot penetrate outside the cable.

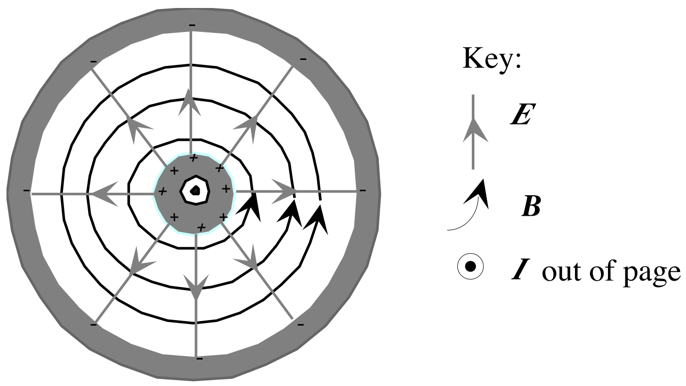

Figure 9. Charge, current and fields in a low-resistance coaxial cable

So the electric and magnetic fields outside the conductors are confined to the space between the conductors and that is where the energy flow must also be. The magnetic field lines form circles. Let's suppose that the resistance of the cable is negligible compared with the resistance connected across the end of it. In that case there will be no axial electric field inside the wire and so the electric field is radial. Now we can see that the energy flux density vector must be perpendicular to both fields, running parallel to the axis of the cable. As both fields drop off inversely with radius, you can see that the energy flux is greatest near the inner wire. (Exercise: If you are happy about writing integrals it is fairly straightforward to prove that the total power going along the space in the cable is VI where V is the potential difference across the ends of the cable.)

Part 5 Conclusion

5.1 Implications For Teaching

Many text and syllabus writers like the motto "Keep it simple ...". Although that is an admirable aim it can conflict with scientific validity. Do we really want to teach stuff that is wrong just because it is simple? I don't think so.

One source of wrong or misleading ideas is the use of inappropriate analogies. Various attempts to make analogies between circuits and water flowing in pipes come to mind. You should be able to see from the account in this article that using water as an analogy for charge does not make much sense. Stories about water and little gremlins carrying buckets of stuff called energy are all best forgotten.

An initial reaction to the story in this article may be that it is too difficult. I would argue that the basic ideas are no more difficult than the concept of field, which already occurs in various school-level courses. In any case field is such a fundamental concept, even more basic than energy, that it is hard to imagine how one can learn any serious physics without it. The arguments about circuits should be no more difficult to grasp than the better known stories told about electromagnetic waves and light – because the basic physics is the same in both cases. I think it is reasonable to expect that high school physics students can grasp the essence of the conceptual framework outlined here.

If you accept the arguments that I have put here, I invite you to help take up the challenge of developing texts and syllabuses that are consistent with the best of current scientific knowledge and avoid gross errors. If the material seems to be too difficult, let's see if we can express it more simply without making it wrong.

5.2 Further Reading

A good account of the role of surface charge and the use of modelling in circuits can be found in the first-year university level text by Chabay and Sherwood (2002) but, strangely, they avoid the whole issue of energy transfer and the Poynting vector (at least they did in the 1995 edition, I have not yet seen the new edition). At a somewhat more advanced level, the great Feynman (1964) gave a challenging account of the Poynting vector in the book of his lectures. Morton (1979) has written an article on the Poynting vector aimed at school teachers and Poon (1986) has written a good tutorial article on energy, energy storage in fields and other matters about circuits (but he does make the odd slip such as referring to the PE of an electron).

5.3 Acknowledgments

Thanks to Brian McInnes, Ann Sefton, Jenny Nicholls and Manju Sharma for useful and constructive criticisms.

Selected Bibliography & References

A much larger bibliography can be obtained from the author on request.

Chabay, R. and Sherwood, B. A., 2002, *Matter and Interactions II: Electric and Magnetic Interactions*, New York, John Wiley & Sons. (ISBN: 0471442550).

Feynman, R. P., 1964, (chapter) Field energy and field momentum, in: Feynman, R. P., Leighton, R. B., Sands, M., *The Feynman Lectures on Physics*, Vol 2, Reading Mass.: Addison-Wesley, pages 27-1 to 27-11.

Jefimenko, O., 1962, Demonstration of the electric fields of current-carrying conductors, *American Journal of Physics*, **30**, 19 - 21.

Jefimenko, O., 1977, Electric fields in conductors, *The Physics Teacher*, **15**, 52 - 53.

Morton, N., 1979, An introduction to the Poynting vector, Physics Education, 14, 301- 304.

Parker, S., 1970, Electrostatics and current flow, *American Journal of Physics*, **38**, 720 - 723.

Poon, C. H., 1986, Teaching field concept and potential energy at A-level, *Physics Education*, **21**, 307 - 316.

Preyer, N. W., 2000, Surface charges and fields of simple circuits, *American Journal of Physics*, **68**, 1002 - 1006.

Rosser, W. G. V., 1963, What makes an electric current "flow", *American Journal of Physics*, **31**, 884 - 885.

Rosser, W. G. V., 1970, Magnitudes of surface charge distributions associated with electric current flow, *American Journal of Physics*, **38**, 265 - 266.

Sommerfeld, A. 1952, *Electrodynamics: Lectures on Theoretical Physics*, Vol III, New York: Academic Press. See §17, Detailed treatment of the field of a straight wire and a coil, pp 125 - 133.