VCE Physics Unit 3: Detailed study –

Einstein's special relativity – the essentials

Key knowledge and skills (from Study Design)

• describe the prediction from Maxwell' equations that the speed of light depends only on the electrical and magnetic properties of the medium through which it is passing, and not on the speed of the source or the speed of the medium;
• compare the prediction from Maxwell' equations of an absolute speed of light with the classical principle of relativity, i.e. no absolute zero for velocity; all velocity measurements are relative to the frame of reference;
• describe Einstein' two postulates for his special theory of relativity and compare them to classical physics
– the laws of physics are the same in all inertial (non-accelerated) frames of reference
– the speed of light has a constant value for all observers regardless of their motion or the motion of the source;
• interpret the results of the Michelson–orley experiment in terms of Einstein' second postulate;
• apply simple thought experiments to show that
–the time interval between two events differs depending on the motion of an observer relative to the events (non-simultaneity)
–length contraction of an object occurs in the direction of its motion when observed from a different frame of reference
• explain the concepts of proper time (t_o) and proper length (L_o) as quantities that are measured in the frame of reference in which objects are at rest;
• explain the unfamiliar nature of motion at speeds approaching c by mathematically modelling time dilation and length contraction using the equations $t = t_o\gamma$ and $L = L_o/\gamma$ where $\gamma = (1 - v^2/c^2)^{-½}$
• apply Einstein' prediction by showing that the total 'mass-energy' of an object is given by $E_{tot} = E_k + E_{rest} = mc^2$ where $m = m_o\gamma$ and so kinetic energy, $E_k = (\gamma-1)m_oc^2$;
• explain that mass can be converted into energy and vice versa, $E = \Delta mc^2$
• explain the impossibility of motion faster than light in terms of relativistic mass $m = m_o\gamma$ at speeds approaching c.

What follows is a summary of the 'essentials' of the curriculum as they relate to the way we have discussed relativity in class and in the text. Not everything in the text is required for the exam, but it is there to help us get a good understanding of relativity – and the better our understanding the better we will find the exam!

• Section 1 Two ideas Einstein did not want to give up:

1) **Galilean** & Newtonian **Relativity**: There is nothing special about zero velocity. Velocity can only be measured relative to some other inertial frame of reference. A force produces a **change** of velocity, not velocity itself (as Aristotle and everyone had thought). That is, a force produces acceleration (a = F/m). Notice that acceleration is **not** relative, but absolute for any *inertial* frame of reference. (An inertial frame of reference is one with a steady velocity.) That is, all observers in inertial frames will measure the *same* acceleration in any frame.

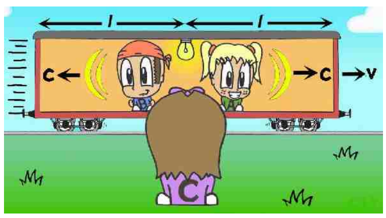
2) **Maxwell**, however, produced electromagnetic equations which suggested that a) light is an electromagnetic wave and b) that all electromagnetic waves travel at a fixed speed of 3×10^8 m/s, regardless of speed of source (not too surprising – sound is same) or of observer (*very* surprising – not like sound). This was quite contrary to the principle of relativity! Maxwell and others thought that this meant there must be an 'aether' which was 'fixed' in space and to which this speed of light must be relative.

Michelson and Morley tried to measure the Earth's speed relative to the aether. Their experiment attempted to measure the difference in speed as the light went across the path of the Earth's motion compared to back and forward along the path. They found no difference in speed with direction of the Earth's motion – as they should have if light really had a speed fixed in the aether. This result could not be explained by the physics of the time. Although it was consistent with the Maxwell prediction about the speed of light, it was quite at odds with the principle of relativity which said velocities were always relative.

Here we need to mention **Newton**, who realised that when we work out relative velocities we are assuming that space is 'straight' and time is 'universal'. This means that space is like a big box with three dimensions (the x, y, z coordinates for example) and that time would be measured exactly the same by all clocks wherever they were. Newton had no reason to believe otherwise, but had the genius to see that this is an *assumption* that we make. These are the assumptions we make when we work out relative speeds (because we add or subtract distances and times). Einstein realised that perhaps this assumption was not correct when it came to large distances and high speeds.

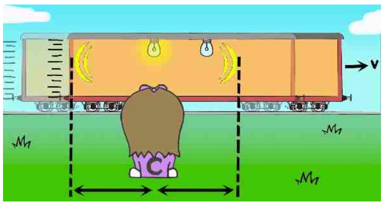
• Section 2 Einstein's crazy idea

Einstein felt that both the **principle of relativity** and **Maxwell's equations** were too 'elegant' and too well founded to be wrong. He therefore explored the consequences of keeping them both, despite the apparent contradiction.



Einstein's postulates were:
- the laws of physics are the same in all inertial frames of reference
- the speed of light has a constant value for all observers

To get a feel for the implications, we follow Einstein and do some 'Gedanken' experiments, notably in our Gedanken train with its flashing light in the middle. The important thing to notice is that the light flashes are travelling at ***c*** for **both inside and outside** observers. This is what is different if we take Einstein's postulates at face value. Also note that while we can't actually 'see' the light flashes travelling (as suggested by the diagram) we can work out where they were, when, so our Gedanken observers do all that for us!



Clearly, A and B see the light reach the ends of the carriage at the same time – and return to them at the same time. But consider the situation as seen by C: As the train moves forward, C sees the backward flash meet the oncoming back of the carriage before the forward flash catches up to the front of the carriage. Clearly then, C sees the light flashes hitting the front and back walls at *different* times, while A and B see the light hitting at the *same* time. So what was simultaneous in one frame of reference was not simultaneous in the other frame.

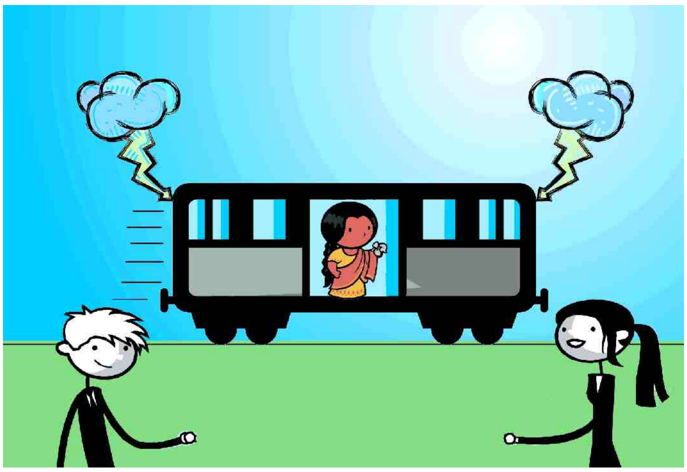
This illustration indicates that **time is relative**. In other words, there is no big universal clock out there somewhere ticking away the 'real time' while all our other clocks do strange things. THERE IS NO 'REAL TIME'. This is hard to get our minds around because we are so used to there being a 'correct' time. Certainly we adjust our clocks for our 'time zone', but that is only a convenience thing so we always expect midday (by the Sun) to occur at around 12 noon. That a moving clock might *inherently* tell a different time to one at rest seems very odd because we never experience anything like that. However, experiments with atomic clocks in aeroplanes and space vehicles have thoroughly verified Einstein's predictions about time. In those situations we are only talking of microseconds or milliseconds, but atomic clocks are accurate to millions of times greater than that. The whole GPS system would literally be miles out if it didn't take into account Einstein's relativistic time.

Now consider the flashes reflected from the front and back walls of the train returning to A and B. All observers must agree that these flashes return to C at the *same time*. It would be totally illogical if C saw the forward and rear flashes reach A and B at different times. Here we need to be clear about what we mean by *events*. An **event** is something that happens at a certain time **and** a certain place. The flashes reaching the front and back walls are two *different* events – they happened in different places and could therefore also happen at different times. The flashes returning to A and B again are *one* event because they happened at the same place. If A and B see them as *one event* then C must also see them as one event. Events which happen at different places must be different events, events which happen at the same place at different times are clearly also different events. Different observers may find different distances and different times between *different* events, but all must agree about what happened in a single event – that is, A and B saw the two flashes arrive as one event, not two.

To get a feel for what this all means, consider the following example. It is a little long-winded, but if you follow it step by step it should help you to understand how events which are simultaneous in one frame may not be in another.

Q.1 Two flashes of lightning just happen to hit the two ends of a fast moving train at the same time. What is wrong with this statement?

These questions are all answered in the following text, but see if you can answer them first before reading on!



Well, yes it is very unlikely! But the real problem is that we have not specified in what frame the events occurred simultaneously. As we now know, if they occurred simultaneously in the train frame they would not have been simultaneous in the ground frame. Let's see why:

Situation 1)

Imagine the flashes were **simultaneous in the ground frame**. That is, an observer on the ground but at the centre of the train (not shown in picture) as the lightning struck, will have seen the flashes arrive together and then deduce, because the distances to the ends were equal, that the flashes must have been simultaneous. The boy, who happened to be at the rear end of the train, will have *seen* the rear flash first, but will be able to calculate (if he survived the lightning) from the delay in *seeing* the other flash (and the length of the train) that the flash at the front end must have occurred at the same time (by his clock) as the one at his end. Alternatively, the boy and the girl (at the front end) could record the times on their clocks and then the two of them could get together at their leisure to compare the times on their two clocks (which are in the same frame of reference). They will find that the two flashes occurred at the same times on their clocks.

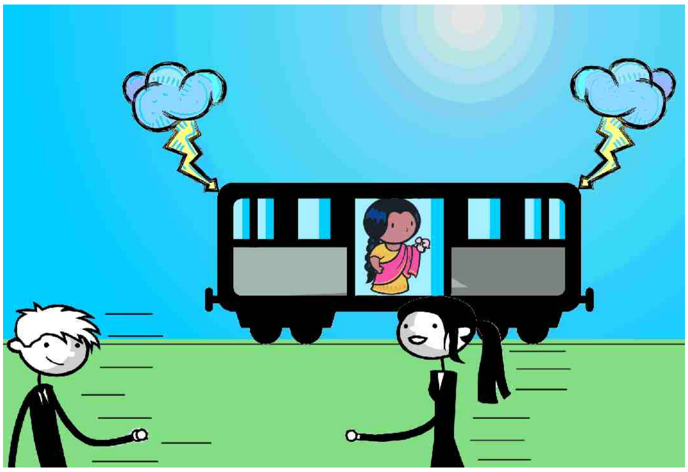
Q.2 But how will the observer **inside the train** see the flashes? Will they be simultaneous? Or which one will be first?

We can answer this question simply by looking at the situation from the ground observers' frame (boy and girl). As the train is moving away from the boy, he will see the light from the rear flash take longer to reach the train observer (in middle of carriage) than the light from the front flash will (it doesn't have to go as far as the train is moving toward it). The boy therefore deduces that the train observer will see the front flash before she sees the rear one. The girl, at the front of the train will deduce exactly the same thing. So what about the observer in the train? She must also agree that the front flash was first. Notice that both sets of observers must agree about what the train observer saw – she either saw one flash first or saw them simultaneously – no observer can see a different *order* for the flashes at *one place*. They can, however, see the time between events (flashes arriving for example) differently. In the train based observer's frame the flashes have come the same distance (half the length of the train) and so (because she saw the front one first) she deduces that indeed the front flash must have occurred before the rear one. So they (the lightning strikes) were *not* simultaneous for her. Time is relative!

We deduced this by arguing from the ground frame, but the train observer must agree that they were not simultaneous. Observers in different frames may measure different times, but they must agree on the order in which any *one observer* saw the events which occurred at the same place (that is, the flashes arriving at the centre of the train).

Situation 2)
What if it was the **train observer** who saw the flashes **simultaneously**? (This was not the same pair of lightning flashes in situation 1.)

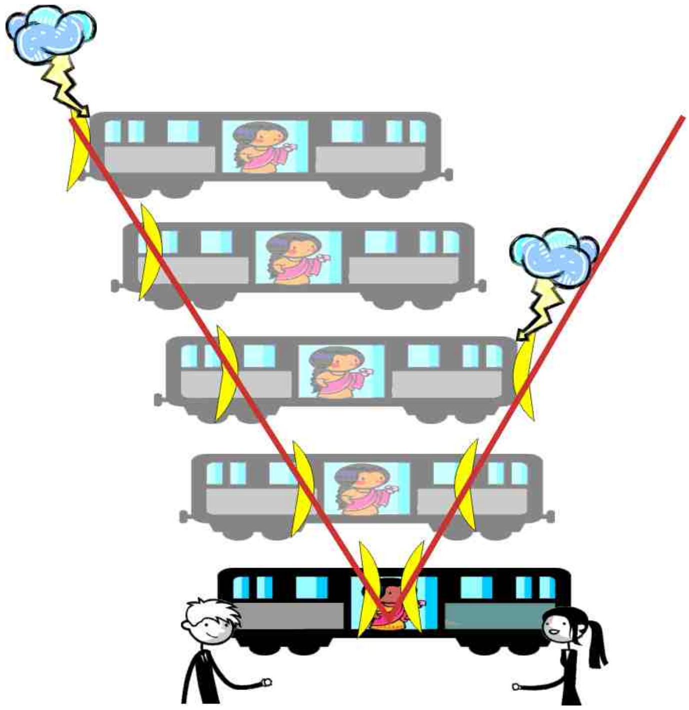
Q.3 If the train observer sees the flashes simultaneously, how did the boy and girl on the ground see the flashes? (Remember that all our observers always see the speed of light as the same.)



Now everyone must agree that the train observer *saw* the flashes together – that was *one* event. So when did the *two* separate events, the two lightning flashes, occur in the ground frame? The train observer says they happened at the same time, but what about the boy and girl? (Try to decide on *your* answer before reading on!)

In this diagram (previous page) we are looking at the situation from the frame of reference of the train – the boy and girl are speeding by in the other direction (to the left). It just so happens (this is the advantage of Gedanken experiments) that at the instant the train observer *sees* the flashes, the girl is opposite her and so also *sees* the flashes together. (Note that neither of them say the lightning flashes were at the same time because they *saw* them at the same time – they know that the light takes time to reach them. The question is how much time.)

The train observer, as we said, deduces that the lightning flashes hit the train at the same time (because they travelled the same distance). But what about the girl? She is moving to the left (as seen by the train observer). In the train frame our observer will deduce that the girl will decide that the left hand flash occurred first (although she *saw* them at the same time).



Q.4 How did the girl deduce that the left hand flash was first?

As the girl is moving past the centre of the train, she *sees* the flashes together. But she will then realise that she must have been closer to the right hand end when the flashes occurred, so if she saw them together the left hand flash must have occurred first because the light had more distance to travel to reach the centre of the train (in her frame).

Here is another way of looking at the same situation:
This time we stay in the **ground frame**. The boy and girl both see the flashes reach the train observer at the same time (just as did the train observer – this is one event). They know that the light is travelling toward the centre of the train at the same speed from both directions. In the diagram this is represented by the V shaped lines and the flashes getting closer to the centre. The train is also moving, but not as fast as the light. We want to know when the light flashes left the ends of the carriages. These are the two points where the flashes coincide with the

ends of the train. The lightning bolts are shown at those events. We can see that the left hand flash must have occurred before the right hand one – because it had further to go to reach the spot where the centre of the train is when the observers all see the light reach the girl in the centre of the train.

Q.5 In this diagram the (ground based) girl was not at the centre of the train when the light reached the train observer. Does that make any difference to our analysis?

The answer is no, because any observer in the ground frame will see the same thing. Depending on how close they are to the centre of the train they may see the flashes (together) reach the train observer at different times, but when they calculate the 'look-back' time they will all deduce the same interval between the lightning flashes and the light arriving at the centre of the train – the left hand interval being longer than the right hand interval.

• Section 3 Time is not what it seems

In order to understand time dilation you need to work through the argument in the text. Unfortunately, on the exam you won't have to explain where the time dilation formula comes from, you'll just have to use it. But to use it properly you need to understand it!

$$t = t_o\gamma \quad \text{where } \gamma = 1/\sqrt{(1 - v^2/c^2)}$$

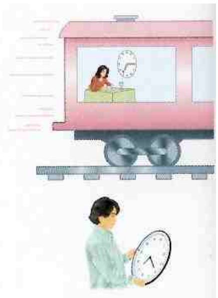
The most important thing in using the time dilation expression is to understand the distinction between relative time and 'proper time'. **Proper time, t_o**, is the time interval between two events as measured by someone in the **same frame as the events**. The relative time, t, is the time as measured by someone moving relative to the frame in which the events occurred.

Some useful values of gamma and speed (it might be worth jotting some of these on your A4 sheet for the exam).

v/c as %	gamma γ
10%	1.005
20%	1.02
66%	1.33
88.6%	2.00
99%	7.09
99.5%	10
99.9%	22.4

Here (diagram below) is another situation that can help us sort out time dilation and the relationships between different events. The train is moving along the track (at relativistic speed!). As it passes the girl (event 1) the clocks in the train and on the ground both read 7:00. As it passes the boy further down the line (event 2) the train clock reads 7:15 and the boy's clock reads 7:20. This is as we expect because time goes slower in the moving frame.

Event 1



Event 2

See if you can answer these questions before looking at the answers which follow.

Q.1 Which of these clocks is reading the 'proper' time? How do you know?

Q.2 From the point of view of the person in the train the world outside is moving by in the opposite direction (to the left). So why doesn't she see the external clock at event 2 reading less than hers (about 7:11) because of time dilation? Or does she?

Q.3 What is the value of gamma in this situation, and so what speed (relative to light) is the train moving at?

The proper time is the time as measured in the frame of reference in which the two events occur at the same place. This is clearly not true of the girl and boy in the ground frame. It is true of the girl in the train however. In her frame the two events occurred at the same place – by her table in the train – but at different times, 15 min apart. So the girl in the train has the proper time.

At event 2, the train passing the boy, the girl in the train and the boy must agree about what they see. They will both agree that the train clock says 7:15 while the boy's clock reads 7:20. Both ground observers must agree about the time recorded from their frame – they can compare clocks and times later. So, no, the train observer does not see the clocks saying anything different to the ground observers. But doesn't she see time going more slowly for the people on the ground? Yes! Relativity is quite consistent in that there is no special frame of reference – the train is as good as the ground. So how come she sees one clock reading 7:00 and the other 7:20? Doesn't that mean that 20 min have elapsed? No!

The boy and girl ground observers are **not at the same place** and so for the train observer, events 1 and 2 were different in **both time and place** as far as the ground frame was concerned. The train observer did not see the ground girl's clock going slow, just that the girl's and the boy's clocks were reading different times. If she could see the ground girl's clock it would indeed read about 7:11 as she passes the boy. The fact that those two clocks would be different in her frame is because the time between different events is different when objects are in motion. Remember that she sees the light from the girl's clock coming at her at speed c, whereas the ground girl will see that same light take longer to reach the train girl (because the train is moving away from the light) – hence when the train clock says 7:15, hers will only read about 7:11. But the train girl seeing the ground girl at 7:15 and the ground girl seeing the train girl at 7:12 are two different events, not the same one, and the two observers will indeed see different times elapse between those two events.

The time in the 'proper' frame (that is, the frame in which both events occurred at the same place – ie, the train carriage) was 15 min. The dilated time was 20 min and so $\gamma = t/t_o$ 20/15 = 1.33. From the table above we see that the speed of the train must have therefore been 66% of ***c***.

What if the gamma value was not on our table? We simply have to use the gamma equation to solve for v! However, it is handy to re-express this equation in terms of v. First, let's make a simplification by defining the speed *relative to light* as V (Capital V). So V = v/c. This means that we can express the gamma equation as:
$\gamma = 1/\sqrt{(1 - V^2)}$. If we square both sides and cross multiply this becomes $1 - V^2 = 1/\gamma^2$
A little more algebra gives: $V^2 = 1 - 1/\gamma^2$ (It is handy to put this on your A4 sheet)

To solve our previous problem we put $\gamma^2 = (4/3)^2 = 16/9$ and so $V^2 = 1 - 9/16 = 7/16$ giving $V = \sqrt{7}/4 = 0.661$, that is V is 66% of ***c***, as we said. Don't forget to 'unsquare' the V^2 to get V when you use this equation – or maybe express it as $V = \sqrt{(1 - 1/\gamma^2)}$.

• Section 4 Time and Space

If time is 'bent' by fast travel, it is not too surprising that space is also. In order to understand situations like the previous train clock and ground clock situations we really need to understand the 'length contraction' as well as the time dilation. Einstein's length contraction equation:

$$l = l_o/\gamma \qquad \text{where } \gamma = 1/\sqrt{(1 - v^2/c^2)}$$

Notice that this time the proper length is **divided** by gamma rather than multiplied by it. Times appear *longer* in the moving frame, and lengths appear *shorter*. Moving objects appear 'shrunk' – but only in the direction of motion, not in the other two dimensions. Again, the 'proper' length is the length as measured by someone in the frame which is at rest relative to the length being measured.

Moving objects are contracted in the direction of motion

Remember that this applies both to objects and distances. As we watch a space ship travelling by at 88.6% of ***c***, where $\gamma = 2$, the space ship will look half as long as normal. But of course the space ship crew will see us looking half as wide as normal also. Furthermore, if we see them travel from the Earth to the Moon (in a couple of seconds) a distance of 380,000 km, they will say they only travelled 190,000 km, BUT that is because the distance between the Earth and Moon to them **is** only 190,000 km, whereas we measure it as 380,000 km. **Space is relative!** The faster you go the more 'shrunk' it appears to be. (Light photons go so fast, space is shrunk to nothing!)

Q.4 The electrons whirling around in the new synchrotron near Monash will be moving at close to 99.99999% of ***c*** where γ is about 2000. The actual length (circumference) of the storage ring is 216 m. How long is the ring in the frame of reference of the electrons? How long do the electrons take to get around the ring (from our frame)?

The apparent length of the ring as seen by the electrons is only about 11 cm! The time to get around is 216 m divided by ***c*** – there is no needed to worry about the 0.000001% difference from ***c***! This is 720 ns (ns = nano seconds = 10^{-9} s).
Q.5 But wait a minute, don't the electrons only see their path as 11 cm? So how come they don't just take 11 cm ÷ ***c*** = 37 ps (picoseconds = 10^{-12} s)? Think how you would answer this question before reading on.

To the electrons the ring is rushing by at 99.99999% of ***c***. And yes it only takes 37 ps for it to go by. However, that is what we expect from relativity. Observers (and even electrons) in different frames see different times between the same events (being at one point on the ring and then being there again).
Q.6 We can work out the time in the electron frame a different way. What is it?

From our frame we see the electron 'clock' going slow by the gamma factor. Divide 720 ns by 2000 and what do we get? Relativity might be strange, but it *is* consistent!

Q.7 Imagine a 500 m long space ship is travelling past an even longer tape measure we have stretched out in space. The tape is at rest relative to us. The ship is travelling at 86.6% of ***c*** (so $\gamma = 2$). We see the front of the ship pass the zero mark on the tape and start our (super fast) stop watch. How long is it before the back of the ship passes the zero mark? And where is the front of the ship at this time? (Again, think out your answer now.)

You will probably have found this question easy enough – the ship appears to be 250 m long and travelling at ½ ***c***, so it will take 417 ns. If someone takes a photo of the ship in front of the tape it will be clear that as the back passes zero the front will be at the 250 m mark. The apparent length has contracted.

Q.8 Ok, the ship took 417 ns to pass the zero point on the tape. And when the back passed the 0 mark the front was at the 250 m mark. Does that mean that the ship is *really* 250 m long? What will the space ship captain think about this?

Let's consider things from the captains point of view. He sees the tape measure flying by at ½ ***c***. And he sees its length contracted. He knows his ship is 500 m (that was its proper length). So when the back of the ship passes the zero mark, where is the front of the ship? At the 1000 m mark! This is because while the ship is 500 m long, the tape appears contracted and so if the captain gets two of his crew to record the positions of the front and back at the time the back passes the zero mark, the front will be opposite the 1000 m mark. That's all well and good, but hang on, didn't we see the front opposite the 250 m when the back passed zero? What's going on?

This highlights the importance of understanding what 'non-simultaneity' really means. The point is that when we said where is the front *at the same time* when the back was at zero we regarded those *two events* (back at zero and front at 250) as simultaneous. But these two events were *not* simultaneous for the space ship captain! The front being at 250 will have occurred *before* the event of the back being at zero. To the captain, it was the front being at 1000 m and the back at zero which were simultaneous.

Notice that both observers in this example can use the tape measure to measure the 'proper' length of the ship – and get the same answer! We actually *know* that the ship only appears to be half its proper length because we know about relativity. So we double the 250 m and get 500 m. The captain knows that the tape measure is going at ½ ***c*** and so knows that it appears shrunk to half its length. So he halves the 1000 m to get 500 m.

• Section 5 E = mc²

This simple little equation can be interpreted (and mis-interpreted) in various ways. What it really says is:
Total energy of an object = relativistic mass × c^2 . But we need to realise that m here is gamma times the rest mass: $m = \gamma m_o$ So $E = mc^2$ can be written as $E_{tot} = \gamma m_o c^2$ Now when at rest $\gamma = 1$, so the total energy of an object at rest is not zero – in fact it is $m_o c^2$ which for an ordinary object is a very big number! Einstein realised that **mass is a form of energy**. And that **energy has mass**.

The kinetic energy, the energy due to motion, is therefore the difference between the rest mass and the total energy: $E_K = E_{tot} - m_o c^2 = mc^2 - m_o c^2 = \gamma m_o c^2 - m_o c^2 = (\gamma - 1)m_o c^2$
As the last expression in this set implies, it can be handy to think of the part of γ above one as representing the kinetic energy. For example: What is the kinetic energy of an electron if its relativistic mass is $3m_o$? Answer: $2m_o c^2$ That is, twice its rest mass energy. (Of course $E_K = ½mv^2$ does NOT give us the correct answer – unless the speed is well below ***c***.)

Q.9 An electron of rest mass 9.1×10^{-31} kg is accelerated in an electric field and given 100,000 eV of energy (1 eV, or 'electron-volt' is the energy an electron gets going through a 1 V battery and is equal to 1.6×10^{-19} J). How fast is it going? First work this out just using Newton's mechanics, then using relativity. Then compare your answer with that below.

Newton says we use the expression $E_K = ½mv^2$ (actually Newton never said this, but it follows from his laws of motion). The energy is $100{,}000 \times 1.6 \times 10^{-19}$ J = 1.6×10^{-14} J Putting this, and the electron mass into the kinetic energy expression gives $v = 1.9 \times 10^8$ m/s. This is well over half of ***c***.

Using relativity, we again know that the kinetic energy is 1.6×10^{-14} J. This enables us to find γ, as $E_K = (\gamma - 1)m_o c^2$
So $\gamma - 1 = E_K / m_o c^2 = 0.1954$ and so $\gamma = 1.1954$

Using our expression for V in terms of γ we find V = 0.548 and so v = 0.548 ***c*** = 1.64×10^8 m/s which is a little over half of ***c*** , and a little less than the value found using conventional mechanics.

This reflects the fact that the mass of the electron at this speed is 1.1954 times its rest mass, and therefore the increase in energy does not speed it up as much as it would if it had no increase in mass. The faster it goes the heavier it gets and so the less acceleration it has. This is why it can never reach the speed of light.

Try repeating this problem starting with a kinetic energy of 1,000,000 eV. Conventional mechanics says the speed should be √10 the previous speed, ie. 6×10^8 m/s, twice the speed of light! See what the relativistic speed is.

It is worth noting that the 1,000,000 eV used here is one million electron volts or 1 MeV. An electron accelerated by 1 million volts would have this energy. Our Van de Graaff will accelerate electrons by about 300,000 V, that is they will have 300 keV of energy. By comparison, the synchrotron will accelerate them to 3 GeV. (A GeV is a Giga eV or 10^9 eV) You can see why the electrons in the synchrotron are going at very close to ***c***!