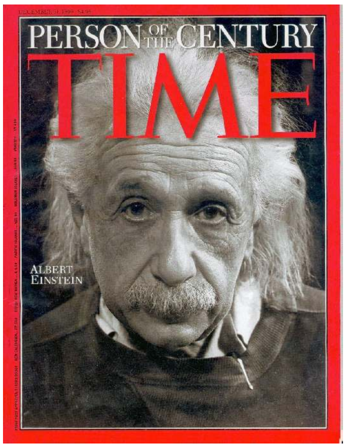
# VCE Physics Unit 3: Einstein’s Relativity DS

PHYSCON Feb 2009 - Keith Burrows
Please note: The PP presentation that goes with these notes can be downloaded from www.vicphysics.org
Follow the links from ‘Teachers’ to Unit 3 Relativity DS

## Why Einstein’s Relativity?

The ***Einstein International Year of Physics*** celebrated what has been called ‘Einstein’s Miraculous Year’. The 1905 publication, not just of his Theory of Relativity, but of three other groundbreaking works as well, was truly a turning point in the history of physics. Relativity is a fascinating aspect of physics, but perhaps the best reason for teaching it at year 12 is that it is an excellent way to give our students a feel for the **real nature of physics**. Most of the great advances in physics (or ‘natural philosophy’) have occurred because a ‘genius’ was prepared to put forward a radical new idea: **Aristotle** – the world can be understood by careful observation, thought and reason. **Copernicus** – the Earth moves around the Sun. **Galileo** – there is nothing special about zero velocity. **Newton** – too many to list! **Faraday** – electricity and magnetism are linked. **Maxwell** – light is an electromagnetic wave. These are but a tiny sample of the natural philosophers who have advanced our understanding by being prepared to look at the world around them in a new way. The fact that we now take these discoveries for granted is testimony to their importance, but at the time they were first put forward, they were totally radical ideas. Certainly Einstein’s ideas were radical when he put them forward – in fact to most of us they still are!



Although there are many, many examples of great leaps in our understanding of the physical world in the twentieth century, the outstanding example is Einstein’s Theory of Relativity. To most people Einstein’s achievement represents the epitome of modern science – and for good reason. He questioned assumptions, he looked carefully at what was known, and he used his imagination creatively, but intelligently to put forward new and radical ideas. He also combined all this with a real concern for the social implications of science as well as being a warm and compassionate human. It is little wonder ‘*Time*’ magazine called him the ‘Person of the Century’.

It had always been assumed (if it was thought about at all) that time and space were ‘straight’, absolute and independent of each other. It is to Newton’s great credit that he realised that this was, in fact, an assumption we make. That it was even a question was something that never occurred to most people. Einstein’s huge leap of imagination was to suggest that not only were time and space *not* ‘absolute, true, immovable’ (Newton’s words), but that **motion through time and motion through space were *interrelated***.

To put it in a nutshell, Einstein said that we are always travelling through ‘***spacetime***’ at a constant rate, but, in a way, we can choose to have more of one and less of the other. We normally travel steadily through time (even if it often seems to be an increasing rate!) but through very little space (on the grand scale of things that is). However, if we could travel at near the speed of light, our travel through space would be at the expense of our travel through time. In fact, light itself travels entirely through space and not at all through time – just the reverse of what we normally seem to do! But we are getting ahead of ourselves, and we are in danger of over simplifying things.

## The purpose of this DS

Einstein’s relativity was (and still is) such a radical idea that it is very hard to really appreciate its true nature. It is important for us to realise that the purpose of this Detailed Study is not to provide students with a comprehensive understanding of relativity – that would be impossible. Rather, it is to give them a feeling for its connection to the rest of the physics they study, and a glimpse of where their physics is headed. Even more important, however, is the insight that it gives us, and hopefully our students, into the **nature and processes of physics**. Too often students see physics as simply as a collection of equations that are used to solve various problems, or maybe as the driving force behind new technology. Important as these are however, they are not ‘real physics’. I suggest that real physics is more a **process** than an outcome – and that this process is wonderfully illustrated by the story of relativity (which actually starts with Galileo).

Is the process of physics more important than its outcomes? This could be debated I guess, but I would suggest that it is *far* more important! Why? Because in the long run it is not the facts, but the ‘feel’ of physics that will be important to the students. There are two reasons:

The ‘facts’ can always be looked up later. It is an understanding of where these facts have come from, how reliable they may be, and how they **relate to the whole picture** that makes the difference between a physicist and someone who just ‘knows’ a lot. “*Imagination is more important than knowledge*” said Einstein. We have no idea where our students will end up and what parts of the story of physics will be important to them. But what will be important is their ability to discern relevant from irrelevant information, to see the difference between reliable sources of knowledge and unreliable ones, and to ask meaningful questions. This is what a ‘feel’ for physics gives us. This is what feeds Einstein’s ‘*imagination*’.

The second reason is that if there is anything that this world needs now (apart from love!) it is a reasoned, thoughtful, and questioning approach to the social, political and environmental challenges we face. The greatest dangers in all these areas are from those who ‘know’ they have all the right answers. And I am not just referring to the various religious fanatics, but to economic and political fundamentalists as well – not to mention a whole host of other groups who can’t see beyond their own immediate dogmas or self interest. The whole story of physics teaches us to keep **asking questions**, to use our reason and our thinking to solve problems, and to look beyond the immediately obvious.

Do we, as physics teachers, underestimate the importance of what we are teaching? More than anything else, our modern western world has been shaped by both the consequences and the philosophy of the ‘enlightenment’, a movement which cast aside authority and dogma for reason and experiment. This was the time of the birth of classical physics. The technological consequences are very obvious, but perhaps we don’t always consider some of the philosophical consequences.

At the end of the nineteenth century physicists thought they had just about tied it all up – the first version of the ‘Theory of Everything’. Their atomic picture of matter and the laws of mechanics and electromagnetism seemed to be able to explain just about everything. (There were of course a few puzzles such as the constant speed and the nature of light, but most people thought a solution was just around the corner.) This sense of certainty, indeed a sense of power, seemed to translate into the mechanistic, materialistic philosophy that drove the twentieth century. Some would say that ‘economic rationalism’, the idea that human interest can be translated into simple economic terms, is its latest incarnation.

Einstein’s papers of 1905 brought that attempt at a theory of everything crashing down. Space, time and light were not at all the simple concepts they appeared to be in classical physics. Matter itself became ‘wavy’ and ‘uncertain’ not long after. Suddenly the world became much more complex – we could say richer and more exciting – than the simplistic pictures of the nineteenth century. Now it doesn’t take much imagination to see that the simplistic social and economic pictures of the end of the twentieth century are due for a shake-up! Just as classical physics heavily influenced the worldviews of the eighteenth to twentieth centuries, perhaps twentieth century physics will have a similar influence on the way we think in the twenty-first century. I suggest that we should hope it does. And that’s why I am looking forward to relativity becoming part of our curriculum.

"*When the ideas involved in relativity have become familiar, as they will do when they are taught in schools, certain changes in our habits of thought are likely to result, and to have great importance in the long run.*"

Bertrand Russell in *ABC of Relativity*

## Who is it for?

I hope that it will be clear from the preceding that I believe this study should be aimed at all of our students (in fact I would say *all* year 12 students!) not just those intending to go on to further physics. Indeed I suggest that the main purpose of this study at VCE level is to give those who will *not* have the benefit of tertiary physics that ‘feel’ for real physics discussed earlier. If only more journalists, teachers, politicians, philosophers – not to mention parents and hairdressers – could experience a way of thinking that is open to new ideas, actively examines the validity of its own concepts, has no place for dogma, constantly searches for ‘truth’, and knows that it does not have the complete picture!

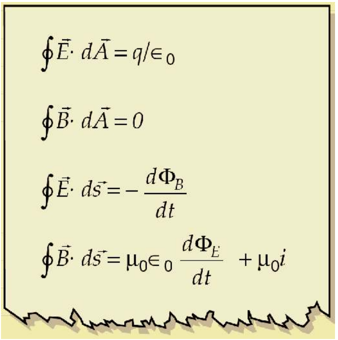
## So where do we start?

While Einstein’s ideas were radical, they were based on the great discoveries of the physicists who came before. In particular he did not want to give up the basic discoveries of Galileo and Newton, as well as those of Maxwell (who could be called the ‘Newton’ of electromagnetism). In presenting this study to students I believe that it is important that they see it as a progression from the work of previous physicists, not as something that ‘overturned’ all our previous ideas. It was actually Newton who said “if I have seen further than others it is because I stood on the shoulders of giants”, but I am sure Einstein would have agreed! And Einstein’s giants certainly included Galileo, Newton and Maxwell. (See *Einstein’s Heros* by Robyn Arianrhod.)

What follows is an outline of one approach to teaching relativity. The underlying assumption is that students will best be able to follow the broad historical development of the ideas. This helps them to see *why* as well as how relativity was developed and enables them to relate it to their own search for understanding. In introducing the theory itself, however, we need to take a simpler approach that looks at the implications of the two postulates in a simple situation, and develops an overview of the theory from there. Please realise that this paper is only meant as a notes summary for the conference presentation and cannot hope to provide a complete treatment. I have developed this approach more fully in *Heinemann Physics 12 (2$^{nd}$ ed.)*.

## 1. Two principles Einstein did NOT want to give up

- **Galileo's principle of inertia** implies that there is nothing special about a velocity of zero, that all velocity measurements must be measured relative to some other object or ‘frame of reference’ (often just called a ‘frame’). A zero velocity in one frame may be a very high velocity in another.
- **Newton’s laws of motion** cannot determine an absolute velocity. A force causes a **change** in velocity, not a particular velocity. All velocities are relative – this is often called the Galilean/Newtonian ‘Principle of Relativity’.
- The **speed of light** was found to be fast but finite (c = 300 million km/sec). Light was also found to be a wave – but there was the question of what was it that was waving, and in what medium did it travel?
- **Maxwell's electromagnetic equations** suggested the possibility of electromagnetic waves which would propagate at a speed fixed by the electric and magnetic force constants. This speed turned out to be the speed of light, c, strong evidence that light is an electromagnetic wave. A problem however, was that the speed did not seem to make allowances for the motion of the observer – something that seemed quite at odds with the principle of relativity!



$$\oint \vec{E}\cdot d\vec{A} = q/\epsilon_0$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\epsilon_0\frac{d\Phi_E}{dt} + \mu_0 i$$

- The equations were also interpreted to suggest that an **absolute frame of reference** (the aether) existed in which light always travelled at $c = 3.00 \times 10^8$ m/s – something also very much at **odds with** the principle of relativity which said there should be no absolute motion. If there was an absolute frame we should be able to detect, for example, the Earth’s motion through it.
- However, experiments, such as **Michelson and Morley's**, failed to detect any motion of the Earth through the aether as it orbited the Sun. Various explanations were offered, but the only one that seemed to make any sense at all was from H.A.Lorentz who suggested that moving objects physically contract, just a little, in the direction of travel – just enough to compensate for the relative motion. However, there was no real basis for this idea. His equations, the ‘Lorentz contraction’ were simply introduced as a way of explaining a very puzzling observation.
- **Einstein** felt, despite the apparent contradiction, that both the **principle of relativity**, as well as **Maxwell’s equations** and their implications for the speed of light, were so **elegant** they just ‘had to be true’.

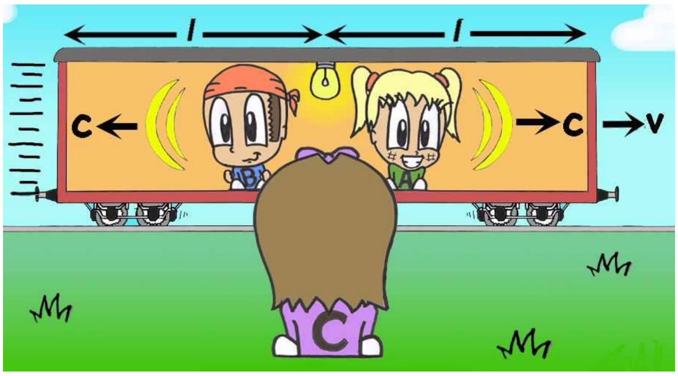
## 2. Einstein's crazy idea

- When just a teenager, Einstein wondered what it would be like to ‘ride a beam of light’. This was a very puzzling question because if we could travel at the same speed we should see the light waves stationary, and that would imply stationary electromagnetic fields which varied with position – something never seen and certainly against Maxwell’s laws. It therefore seemed that it should **not be possible to reach the speed of light**! It also hinted at the constancy of the speed of light for all observers.

- He also decided that **Galileo's principle of relativity** (as extended by Newton) was so elegant it simply had to be true. In brief, this says that there is nothing special about a velocity of zero – all velocity measurements are relative. It also implies that space itself has no ‘centre’ or ‘edges’ or any other way of fixing a frame of reference – and therefore no possibility of an absolute velocity.
- Einstein was also convinced that **Maxwell's electromagnetic equations**, and their predictions were sound. In order to keep both the principle of relativity and Maxwell’s equations he put forward two postulates and looked at the consequences of accepting them. His two postulates of '**special relativity**' were (in effect):

I. **No law of physics can identify a state of absolute rest**.
II. **The speed of light is the same for all observers**.

- Simple statements, but if we think about them, the second seems to be inconsistent with the first. If there is no absolute frame, all velocities should be relative and hence the speed of light should depend on the observer. Einstein realised that accepting both of them implied that there was something wrong with our normal ideas of space and time. He said that **space and time were not absolute** but related (that is ‘relative’) in a four dimensional universe of ‘**spacetime**’. (It is notable that Newton had realised we normally accept the existence absolute time and space, but that there was no fundamental justification for this.)



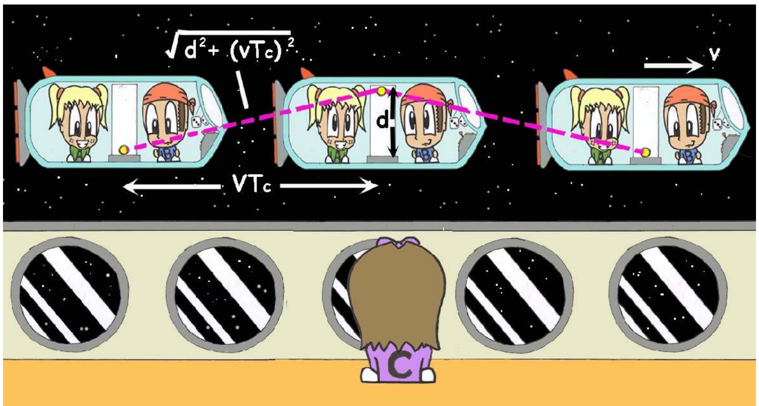
- If the speed of light is constant for all observers, it turns out that an inevitable consequence is that two events which are simultaneous in one frame of reference are **not necessarily simultaneous** in another – as illustrated by the ‘flashlight in the train’ example – the flash from the centre reaches the ends at the same time for those inside but different times for outside observers as the train has moved toward the rear directed pulse but away from the forward directed pulse. (Note that this is not a result of the time taken to get from the ends to the observer – that has to be taken into account.)
- If two events can be simultaneous in one frame but at different times in another, this implies that **time** as measured from different frames of reference might not be the same. It therefore appears that time and space are interrelated in a four dimensional (one time and three space) universe of **spacetime**.

## *3. Time is not as it seems: Time Dilation*

In order to find a quantitative relationship between the time measured in one frame of reference and that measured in another we can look more closely at the flashes in the train:

- The length of half the train is l, its speed v and the speed of light c. The time for the light pulse to reflect from the ends and back to the centre is then $2l/c$ (for the observers inside).
- The observer outside sees things differently – unlike the situation with balls or sound! The light has a velocity of c (**not** c + v) and this time we find that the time for the light pulse to reflect from the ends and return to the centre is $2l/c \times 1/(1 - v^2/c^2)$. That is, different by the factor $1/(1 - v^2/c^2)$, which we call $\gamma^2$. But note carefully that this assumed that we can cancel the l’s!

To be a little more careful about the change in time we need to look carefully at what we mean by a ‘clock’. The simplest clock to imagine (but not to build!) is the ‘light clock’ in which we count the rate at which pulses of light bounce back and forth. The advantage of the light clock is that being based on light we know it will take any relativistic effects into account. Picture a light clock in a space ship, oriented so the light travels perpendicular to the motion of the ship.



- The pulses in a light clock in a moving frame of reference have to travel further when observed from a stationary frame. But remember that the light still travels at the same speed and so it will take longer for each pulse – so this effectively means that time appears to have slowed in the moving frame (as observed from the stationary one):

The height of the clock is d, and the speed of the ship v, so looking at the zig-zag path of light, and assuming that light always travels at c, we relate the distance of each ‘zig’ to the speed of light and the apparent time for this zig ($T_A$ in the moving frame, $T_C$ in the stationary frame):

From within the moving frame, A: $d = cT_A$

From the stationary frame, C, the ship moves a distance $vT_c$ in one zig and the light travels the hypotenuse, $cT_c$ of a Pythagorean triangle where: $d^2 = c^2T_c^2 - v^2T_c^2$

Putting these together (with a bit of algebra) we get: $T_c/T_A = \gamma$ where $\gamma = 1/\sqrt{(1 - v^2/c^2)}$, the Lorentz factor.

- This is Einstein’s famous **time dilation equation**; $\mathbf{t = \gamma t_o}$ , which relates the time, t, in a moving frame to the same interval $t_o$ in the observer’s frame. Needless to say, it has been found to apply to a very high degree of accuracy whenever we observe time in a moving frame of reference. However, note that even at 10% of c the Lorentz factor only makes about 0.5% difference – but with extremely accurate clocks the difference is measurable even for commercial flights around the Earth. (And of course GPS systems need to take it into account.)
- Of course we can reverse the process and look at the situation from the space ship observer’s point of view in which case s/he sees the ‘stationary’ observer’s time slow down. This may seem to be contradictory, but in fact is not, because there is no ‘real’ time for comparison. All time is relative!
- Now if one observer decides to go and meet the other to see who ‘really’ slowed down, what will we find? In order to do this one of them will need to accelerate (slow down, turn around and speed up again). This makes the situation non-symmetric and this is why we don’t have a paradox. In fact, the one who accelerated turns out to have experienced less time. (We come to the ‘twins paradox’ shortly.)

## 4. If time is strange, what about space?

• In the earlier train result that the outside observer ($T_c$) saw the time for flashes to return to the centre of the carriage as $T_c = T_A\gamma^2$ we cancelled the l's in the two equations. The 'light clock' (and Einstein's postulates) tells us that the time should be dilated by $\gamma$ rather than $\gamma^2$. The extra $\gamma$ factor results from the fact that the length of the train was dilated by $\gamma$ in the outside frame of reference and thus the length l of the train was actually shorter as measured by the outside observer.

• So the special theory of relativity says that time and space are interdependent. Motion shortens space in the direction of travel but not at right angles to it. Thus a moving object will appear shorter, or appear to travel less distance, by the factor $\gamma$ Einstein's **length contraction equation** is therefore: $\mathbf{l = l_o/\gamma}$ where $l_o$ is the so called *proper distance*, just as $t_o$ is referred to as the *proper time*, that measured by an observer in the same frame of reference as the two events.

• Einstein said that we live in a four dimensional world of *spacetime* in which space and time are interdependent. Our motion through space comes at the expense of motion through time.

• It may seem all very theoretical – after all time in the moving frame only seems to slow down to an outside observer, not those in the frame. However, Einstein's famous so called 'twins paradox' illustrates the point that it is a practical, and testable, theory:

*Imagine that one of a pair of twins takes off on a long space journey, say to Vega, 25 light years away. If we measure the speed of the spacecraft at 99.5% of c ($\gamma = 10$) it will take just over 25 years as measured from Earth. But this same time interval as measured in the spacecraft will be only 2.5 years (ie., 25/$\gamma$). Remember that the traveller will see the galaxy flashing by at 0.995c and hence it will be contracted by a factor of 10. Now imagine that our traveller stops, doesn't like the Vegans, so turns around and comes straight back – a real traveller would have been totally flattened by the acceleration, but this is a 'Gedanken' traveller! For the traveller, it takes another 2.5 years to get home, but when he arrives home he finds his twin 50 years older compared to his own 5 years.*

In fact this is no 'paradox'. There is nothing self-contradictory about it, it is just well outside our normal experience! While we can't do this sort of experiment, we can do similar things with highly accurate clocks in orbits around the Earth. Einstein's predictions are always found to be as precise as can be measured. Weird maybe, but not a paradox!

## 5. Faster than light? Momentum, Energy and $E = mc^2$

• Because the Lorentz factor, $\gamma$, approaches infinity at the speed of light, the **length** of a moving object approaches zero and **time** comes to a standstill as the speed approaches c.

• As well, relativistic **momentum** also includes $\gamma$ and hence as more impulse is added, the momentum increases but most of the increase is reflected in the $\gamma$rather than in the speed. It is as though the mass seems to increase toward infinity as the speed gets closer to c. So what happens *at* c? Einstein said that these things, along with the electromagnetic nature of light, suggested that the velocity of light was, in fact, a natural speed limit. No mass could ever be accelerated right up to c. Only massless light photons could travel at c. In fact, that is the only velocity they *can* travel at. When they are absorbed they vanish, leaving only their energy and momentum.

• Einstein was able to show that the kinetic energy of an object was given by the rather odd expression: $E_k = (\gamma - 1)m_oc^2$ which at normal velocities reduces, surprisingly enough, to the familiar $E_k = \frac{1}{2}mv^2$ . However, another way of looking at this expression is: $\gamma m_oc^2 = E_k + m_oc^2$. Einstein interpreted the $\gamma m_oc^2$ as the 'total energy' of the body which is equal to the kinetic energy plus what he called the 'mass energy', $m_oc^2$. This mass energy was somehow associated with the mass of an object.

• This we now refer to as 'mass-energy equivalence', and we all know that the realisation that there was a huge store of energy associated with mass eventually led to the release of some of this mass-energy in the nuclear bomb and in power reactors.

• The commonly used expression $E = mc^2$ (which is more properly written $E_{tot} = \gamma m_oc^2$) actually refers to the total energy, which includes the kinetic energy – but of course that is a very minor part of the total.

• In any process that releases energy, the mass associated with that energy is also released. It is not that mass is 'converted' to energy, it is more that energy has mass, and that mass is 'lost' with the energy released. If we could enclose an atomic explosion in a very strong box (!) we would find that the mass of the box did not change after the explosion – the energy is still there.

• A vast amount of experimental evidence now supports Einstein's special theory of relativity.

• Magnetism can only be understood in relativistic terms. (Otherwise why would two parallel currents attract when viewed from the frame which is travelling at the same speeds as the currents?!) See VicPhysics for a fuller explanation of this - soon. Although this is not in our syllabus, it is a fascinating illustration that relativity doesn't only apply to high speed rocket ships, but also to slow moving electrons.

Comments and helpful suggestions welcome keithphysics@optusnet.com.au
Keith Burrows
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