Luna Park Physics

Why are Luna Park rides 'fun'? The answer, of course, is that it is all in the physics! In particular, most of the fun comes from experiencing different, and changing, 'g-forces'. So what do we actually mean by 'g-forces'? It is tempting answer this question by saying that they are 'gravity forces'. Wrong! In fact we will never (unless we become an astronaut and travel well away from the Earth) experience any changes in the force of gravity on us. Whatever we do, a constant force of gravity ($\mathbf{F_g} = m\mathbf{g}$) acts on us all the time. And the value of **g** stays very much the same at 9.8 N/kg. So 'g-forces' are *not* the force of gravity on us. What, then, are they? Let's consider some simple situations where we experience 'g-forces' before considering the more complicated ones we experience at Luna Park.

Simple gravitation and reaction forces

As you sit still on a chair, think about the forces you can actually feel acting on you. The most obvious is probably the upward force from the chair which is holding you up. There will also be an upward force from the floor on your feet, and perhaps from your desk on your arms. If you raise an arm you are probably conscious both of the muscle force in your biceps and of a force pulling your arm down. That downward force is of course the force of gravity. In fact you are probably conscious of the force of gravity pulling downward on your whole body. Think for a moment, however, about why you are conscious of that downward force. Take away the chair and desk, as well as the floor, and you would not *feel* the force of gravity – although you *would* notice the world moving upwards at an increasing rate! So it is not really gravity we feel, but the **reaction** forces around us which prevent gravity from pulling us further and further toward the centre of the Earth.

Why do we call them 'reaction' forces? Simply because they are forces which *react* to objects which exert forces on other objects. Our bottom pushes on the chair, and as a result the foam in the chair compresses and pushes back on us – until the two forces are equal and we stop compressing it any further. In other words, the foam *reacts* to the force we exert on it by pushing back with an equal force. We call this force a 'reaction force', which we can write as $\mathbf{F_R}$. (Sometimes, because a reaction force is usually at right angles it is called a 'normal' force and written as **N**.) If we are standing it is the carpet, or even the concrete, which compresses (if ever so slightly) to produce the $\mathbf{F_R}$ force which stops us from falling toward the centre of the Earth. In normal circumstances (but certainly not all circumstances) the reaction force equals the force of gravity (actually $\mathbf{F_R} = -\mathbf{F_G}$) and we remain in equilibrium.

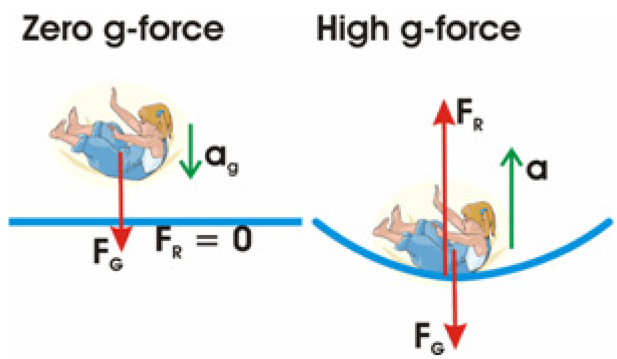
It is worth noting here that, as with *all* forces, they come in pairs: what we often call Newton's 'equal and opposite action–reaction forces'. His third law tells us that these forces are *always* equal in magnitude and opposite in direction. It is important to remember that when we talk about the 'reaction force' $\mathbf{F_R}$ being (normally) equal to $-\mathbf{F_G}$ these are **not** an action–reaction pair. The reaction force $\mathbf{F_R}$ that we are talking of when sitting on a seat is one of the seat–bottom pair of forces we could call $\mathbf{F_{BS}}$ and $\mathbf{F_{SB}}$, the force on our bottom from the seat and the force on the seat from our bottom. Newton's third law tells us that $\mathbf{F_{BS}} = -\mathbf{F_{SB}}$. The $\mathbf{F_R}$ we have been discussing is of course the $\mathbf{F_{BS}}$ force. While $\mathbf{F_{BS}}$ and $-\mathbf{F_{SB}}$ are *always* equal, $\mathbf{F_R}$ is only equal to $-\mathbf{F_G}$ when we are in equilibrium on the seat. Remember that an action–reaction pair always act on two *different* objects (A on B, and B on A) while the $\mathbf{F_R}$ and $\mathbf{F_G}$ we are speaking of are both acting on the *same* object, us.

Actually it is not hard to take away these reaction forces and experience gravity alone – simply jump off a table. While we are in flight we are experiencing only gravity (and maybe a little air resistance). A better way to experience this is to jump on a trampoline. As soon as we leave the mat, whether going up or down, we are experiencing only the force of gravity (leaving aside the small air resistance force). As you jump on the trampoline, and if you can put your mind to it, think about the forces you experience while in the air. You will probably conclude that you can **not** *feel* gravity at all. Certainly you can *see* its effects, but what you *do* notice mainly is that you can not feel the usual reaction forces – until you hit the mat again!

While you are on the trampoline, also think about how you know when you are at the top of your jump. It is clear enough if you are watching the surroundings, but shut your eyes for a moment – **but not if you don't feel safe!** Apart, perhaps, from a slight sensation of the wind changing direction, you can not tell when you are at the top. There is no change of force, or acceleration, at that point – only a continual (downward) **change** of velocity. In fact we are in **free-fall** all the way through the jump from the moment we leave the mat until we touch it again. The expression 'free-fall' simply means that the only force acting on us is gravity and so we are accelerating at 9.8 m/s².

So what are g-forces?

Simply put, what we mean by g-forces are the **total reaction forces** acting on us at any time, expressed in terms of the normal gravitational force (**F_G** = m**g**) we experience. In the normal course of events we experience '1g', that is, a reaction force equal to the force of gravity acting on us. This reaction force might come from the floor or a seat, or whatever is holding us up. But notice that it is a physical force which comes from some *thing*. It is not the force of gravity. If, for example at the bottom of a playground swing, we experience a total reaction force of 2mg, this would be referred to as a '2g' force. This would of course mean that the *net force* on us is 1g upwards (that is, the 2g upward, minus the 1g downward from gravity). As a result (as Newton's second law tells us) we must have an acceleration of 1g (that is, 9.8 m/s²) upwards – which is provided by the ropes of the swing. (Think about our different motion if they suddenly broke!)



Back on our trampoline, as soon as we leave the mat, nothing can exert any reaction force on us and so here we are experiencing *zero g-force*. When we land back on the mat, the mat exerts an upward force on us which increases with the amount of compression. Just after we touch it, it will exert a small upward (reaction) force which is less than our weight. We experience *low g-force* for a moment. As we sink further into the mat the reaction force increases until it is equal to, and then greater than our weight. At the point of maximum compression we are experiencing a reaction force which will be several times our normal reaction force, that is, we experience *high g-force* at that point. This is what slows us down and then accelerates us upward again – at quite a bit more than 9.8 m/s² at the point of maximum compression. If we allow ourselves to stop, the mat oscillates up and down for a while until it comes to equilibrium when the reaction force is again a steady *one g*.

It is worth noting that we tend to use an expression like 'one g' in two different ways. Sometimes it refers to the 'g-force', or reaction force as in our discussion of the forces we experience on the trampoline, and sometimes to acceleration. So that as we fly above the trampoline mat we experience zero *g-force*, but at the same time have an *acceleration* of one g in free fall. The context of a statement should make the meaning clear, but it is important to distinguish between these two types of use.

We could say that the whole point of Luna Park is to put us in situations where we experience different, and varying, g-forces. Of course we could add that there is more to it than that! The aim is also to create an atmosphere where we can enjoy (to varying degrees probably!) these changes. But the fundamental ingredients of a fun park are devices which put us in situations which cause us to experience changing g-forces. Let's look at some types of rides and the g-forces involved. As the physics involved is a little different, we will look first at rides which are more or less straight, and then at rides which involve curves of various sorts.

Straight (more or less) rides

A ride such as the Scenic Railway basically goes along a mostly straight track which rises and falls with varying slopes. (We will deal with the curved sections shortly.) The Coney Drop drops its occupants straight down. Other rides, such as the Dodgems, also move in more or less straight lines with rather sharp changes of course.

The first thing to notice about these rides is that the straight sections at constant speed are not very interesting. It is the *changes* in motion that give us the thrills! In some ways this is an illustration of one of the very important principles of physics: **Galilean relativity**. Galileo realised that there was no real difference between motion at a steady speed in a straight line and no motion at all. In fact he said that the only difference was the frame of reference from which we choose to make our measurements. Galileo realised that it is *acceleration*, not velocity, which resulted from a force. A little later, Newton expressed this principle in his famous second law: $\mathbf{F}_{net} = m\mathbf{a}$. That is, acceleration is a direct result of the net, or sum total of all the forces acting on us. In the Luna Park rides it is this relationship between acceleration and force which makes them interesting!

Two ways to find forces in a horizontal collision

A simple case of rapid acceleration occurs when a Dodgem car runs straight into the barrier. Here we have a situation where the net force is basically equal to the force between the bumper on the Dodgem and the barrier, both of which are 'springy'. The force here is not constant, it will increase rapidly with the amount of compression of both the bumper and barrier. However, if we can estimate the amount of compression, and know the mass and speed of the Dodgem we can find a figure for the average acceleration (from $v_f^2 = v_i^2 + 2ad$) and hence an average force. As springs are usually fairly linear in their F – x characteristics the maximum force is probably about twice the average force.

An alternative way to find the same result is to use energy considerations. We can work out the approximate kinetic energy of the Dodgem and rider from estimates of speed and mass. If the Dodgem comes to a halt in the collision, most of the kinetic energy will have been converted to potential energy in the spring bumper and barrier. Given an estimate of the maximum compression we can then find the stiffness constant of the spring (as the spring potential energy $U_s = \frac{1}{2}kx^2$) which in turn enables us to find the force at various estimates of compression.

What does the rider experience in this collision? The answer of course is a fairly high, if momentary, g-force. To make an estimate of this g-force we need to consider the appropriate reaction forces. We can assume that the distance over which the driver of the Dodgem is brought to a stop is similar to the total compression of the bumper and barrier (we could also add an estimate due to the flexibility of the seat belt). In a similar way to that in which we calculated the force on the car, we can find the average net force on the driver (from the acceleration and mass). Dividing this force by mg gives us the net force as a g-force. This is of course the horizontal component of the total g-force – in this situation we should add (vectorially) the vertical 1g reaction from the seat in order to obtain the total g-force experienced.

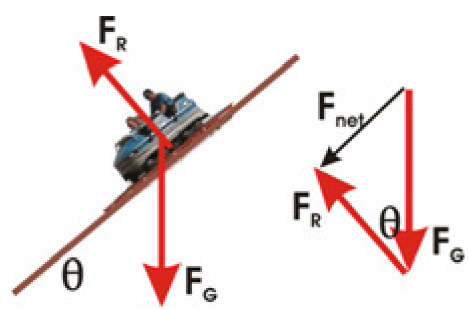
You probably realised, in the last example, that all we really needed was an estimate of the acceleration of the driver of the Dodgem. In that example we actually multiplied by the mass to get the force and then divided by it again to get the horizontal g-force. However, note that this is fine provided we are only interested in the horizontal g-force. This is because gravity has no horizontal component and so there is no steady horizontal g-force to be taken into account. Because we normally experience a constant 1g upward g-force we can not equate a vertical g-force directly with the vertical acceleration and so for any motion involving vertical acceleration we have to be more careful to find the actual forces involved.

Vertical motion

The Coney Drop involves free fall followed by a sharp deceleration. Whether we are jumping on the trampoline or dropping in the Coney Drop the physics is the same – we are in free fall. That is, the only force acting on us is gravity (apart from our possibly clutching at the safety bar!). This means that while the cage is dropping we are experiencing a zero g-force (but 1g of acceleration). Then the cage motion has to be arrested and we slow down with considerable acceleration and experience a high g-force. As we slow, the reaction force from the seat is found from Newton's law, $\mathbf{F_{net}} = m\mathbf{a}$, where the $\mathbf{F_{net}}$ is the vector sum of gravity $\mathbf{F_G}$ (= $m\mathbf{g}$) downward and $\mathbf{F_R}$ upward. Taking upward as positive the equation can be re-written in non-vector form as $F_R - mg = ma$, and so the magnitude of F_R is given by $F_R = ma + mg$, which we can see as the force needed to accelerate us upwards plus the force needed to balance gravity. To express this as a 'g-force' we divide by mg and so we find that the g-force is given by $(a + g)/g$.

For small acceleration this approaches 1g as we expect. Note, however, that if the cage slows with an acceleration of g, the g-force we experience will be 2g. We could of course say that one g is due to the acceleration and one g is due to gravity.

Notice that we could also show that in free fall we experience zero g, as the equation above, giving the magnitude of F_R, would simply become $F_R = ma - mg = 0$ (as a = g and the directions are the same).



The roller coaster (Scenic Railway) involves free acceleration down a track inclined at some angle to the horizontal. At one extreme this becomes simple constant horizontal motion (where we experience the usual 1g) and at the other extreme it becomes free fall (0 g-force). If the coaster is rolling down the slope with little friction (which is *not* always the case) our usual inclined plane analysis tells us that $F_R = mg\cos\theta$ where θ is the angle from horizontal, and $\mathbf{F_R}$ is normal to the track. Clearly then, when we divide this equation by mg, the g force we experience on this slope is simply equal to cos θ. We have talked of free acceleration down the slope, but of course the same is true as the coaster slows down when it comes to an uphill slope.

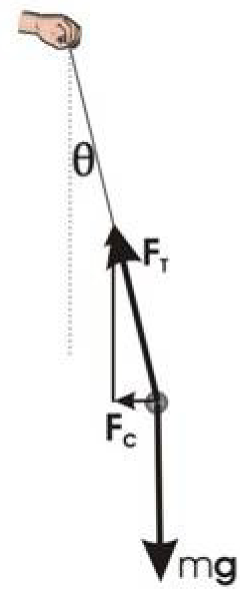
Rides with curves

Most of the interesting rides involve curved motion – either in the vertical or horizontal plane (or both). A full analysis of the acceleration involved in a curved path is well beyond our requirements, however, in many situations we can approximate the curved motion to a section of uniform circular motion. The acceleration of something moving in a circle is given by $a = 4\pi^2R/T^2$ or $a = v^2/r$. For rides where we have uniform circular motion (for example the Merry-Go-Round) and we can easily measure the radius (R) and period (T), the first of these equations is simple to use. Many rides do not have uniform circular motion however, but they do have sections where we can approximate the motion to uniform circular. For example, the roller coaster at the top or bottom of a hill. At the top or bottom the acceleration along the track (the *tangential* acceleration) is close to zero while the *radial* acceleration is reasonably constant, and equal to v^2/r.

Horizontal circles

On the Merry-Go-Round we feel as though we are being flung (gently) outwards, because your body will continue in a straight line along the tangent, unless acted upon by a force to push or pull you inwards towards the centre. You are moving in a circle, so the acceleration is inwards towards the centre of the Merry-Go-Round and it is called a *centripetal* acceleration. Because the direction of the acceleration is inwards, the direction of the net force must also be an **inwards**. This is provided by the horse we are riding or the post we are holding on to.

It is easy enough to calculate the magnitude of this centripetal force F_C from the acceleration determined from the radius and period of the Merry-Go-Round, but it is interesting to compare this to the force as actually measured. To do this we simply measure the angle, θ, of a pendulum bob hung so that it is free to move radially outward. The two forces acting on the pendulum are gravity (m**g**) and the tension in the string ($\mathbf{F_T}$). The vertical component of $\mathbf{F_T}$ balances gravity, but the horizontal component provides the net force $\mathbf{F_C}$ to keep the bob going in the circle. As can be seen in the diagram, the tan of the angle θ (from the vertical) is simply the ratio F_C/mg, so the magnitude of $\mathbf{F_C}$ is given by $F_C = mg \tan\theta$. To express this as a g-force we divide by mg and so the g-force is simply equal to tanθ.



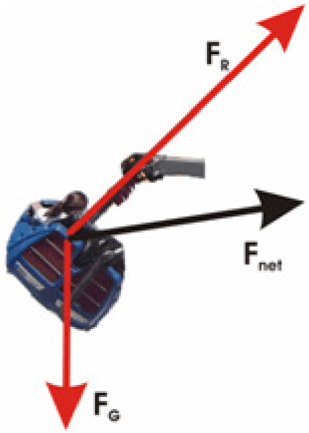
An alternative way of measuring the centripetal acceleration is to use an accelerometer. This could be the electronic variety or the type which use a weight which moves in a curved tube – this is simply a variation on our pendulum bob and the physics is the same.

So what g-force does our rider experience? The answer to this question is sometimes given simply as the centripetal component (tanθ) as calculated above, but more correctly, as the vector sum of this component and the 1g downward weight force. That is simply equal to the tension force in the string shown as $\mathbf{F_T}$ in the diagram and equal to mg/cosθ. In g-force terms that is simply 1/cosθ. As we have said before, the g-force is simply the total 'reaction force' acting, which in this case is the just the tension in the string. It is important to make clear in the context of a statement about g-forces whether we are referring to the total g-forces or simply the centripetal component of them.

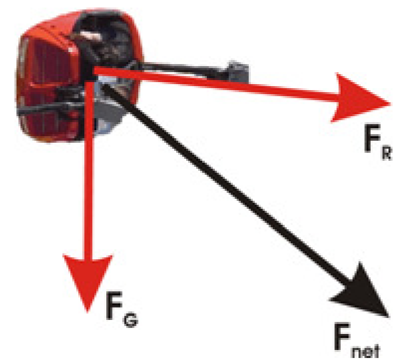
Most physics students will want to experience greater g-forces than those found on the merry-go-round! One appropriate ride is aptly named 'Power Surge'. On this ride, carts which are suspended from a rotating structure are free to swing outwards as the disk rotates. (We will assume horizontal rotation for a moment.) They are, in effect, behaving just as our pendulum bob in the diagram above. The total g-force experienced by the riders is then simply equal to 1/cosθ as explained above. No need for an accelerometer – the cart is its own accelerometer! It is interesting to compare the g-force found in this way with that calculated from the period and radius of the ride (which gives us the centripetal acceleration). Don't forget to add the centripetal and gravitational forces though. (It would be necessary to obtain an *average* measurement of the angle θ for reasons explained in the next paragraph.)

Now in fact the Power Surge ride is not simply horizontal. As it rotates, the axis is tilted from the vertical, and this tilt rotates around vertical with a period similar to the circular motion. This means that each cart is oscillating up and down as well as having circular motion. At the top of the cycle then, as well as centripetal acceleration there is downward acceleration and at the bottom there is upward, as well as centripetal, acceleration. To analyse this motion fully is obviously beyond our requirements, but we can discuss some of the reasons for the feelings the riders experience.

There are two significant forces on the carts (and riders): gravity $\mathbf{F_G}$ and the reaction force $\mathbf{F_R}$ exerted through the supporting rods as shown in the diagram at left. This diagram represents the situation near the *bottom* of the vertical motion. $\mathbf{F_G}$ and $\mathbf{F_R}$ add to give the net force $\mathbf{F_{net}}$ which, in this case, is a little above horizontal. The horizontal component of this force is the centripetal force and the vertical component is lifting the cart.



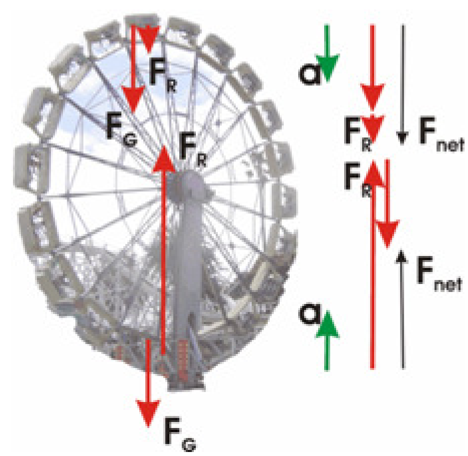
The diagram at right shows the situation near the *top* of the vertical motion. In this case the cart may even be above the horizontal, that is, $\mathbf{F_R}$ has a downward component – at this point the cart is accelerating downward (whether still actually rising or falling) as well as inward.



It is worth noting, again, that in these two diagrams it is $\mathbf{F_R}$ which the riders experience as the 'G-force'. At all times they experience only a perpendicular force from the seat, but that force varies, which is what makes the ride more exciting! As we can see from the two diagrams, $\mathbf{F_R}$ is greater when the cart is at the bottom of its vertical motion and smaller at the top of it. This is because gravity is working against $\mathbf{F_R}$ in the lower half of the vertical cycle (notice that $\mathbf{F_{net}}$ must have an upward component in that half) but with $\mathbf{F_R}$ in the top half of the cycle, to give $\mathbf{F_{net}}$ a downward component.

Vertical circles

One of the most 'exciting' rides is the Enterprise, a disk which rotates initially in the horizontal plane but then becomes vertical so that the occupants are upside down at the top. While the motion is uniform circular motion with a constant inward acceleration, this acceleration results from different combinations of forces at different places around the circle. While an analysis of the g-forces all around the circle is complicated, at the top and bottom the reaction forces are parallel to gravity and it is easy to see what is happening. As can be seen in the diagram, at the top the gravitational ($\mathbf{F_G}$) and reaction forces ($\mathbf{F_R}$) act in the same direction to add to give the net force ($\mathbf{F_{net}}$) which results in an acceleration **a** downward. At the bottom the gravitational ($\mathbf{F_G}$) and reaction forces ($\mathbf{F_R}$) act in opposite directions and so $\mathbf{F_R}$ needs to be much greater in order to produce an upward net force ($\mathbf{F_{net}}$) which results in an acceleration **a**, of the same magnitude, upward.



Again, the g-force experienced by the passengers is $\mathbf{F_R}$, which in this ride varies from very little at the top, to 2 or 3 g at the bottom. (Notice that if $\mathbf{F_R}$ was zero at the top, it would have to be 2g at the bottom.) This, combined with the world apparently circling around one, is the reason for the rather strange sensations one experiences during and after the ride! It is interesting to close one's eyes for a few cycles. In this case we don't see the world spinning, but only feel the varying g-force – it is a lot easier to take!