Einstein's Train and other 'Gedanken' experiments

In his book *Relativity: The Special and the General Theory* Einstein uses a 'thought experiment' involving lightning flashes striking a railway line at two different places, A and B, *simultaneously* to illustrate the difficulty of defining this concept. Since his use of this 'Gedanken' (German for thought experiment) many 'popular' explanations of relativity have copied him. However, not all of these popular accounts are consistent with Einstein's – or each other. Much of the confusion centres around the 'look-back time' issue and so it is important to be clear about this. We illustrate this with Einstein's original Gedanken experiment and then some other versions.

How do we know if two events are simultaneous? Einstein says we must agree on a method of determining simultaneity before we can talk of two events being simultaneous. Suppose we carefully measured the distance AB along a railway embankment and then placed a suitably equipped observer M at the mid point of AB. If that observer sees flashes at A and B at the same time then we can say that they must have been simultaneous – because the light would have taken the same time to travel from both A and B to M. However, note that observer M, let's call him Mike, saw the flashes a little after they actually occurred in his frame. We can call that time the 'look-back time'. It is simply equal to d/c where c is the speed of light and d the distance from M to A or B. Also notice that when we say that the observer 'sees the flashes' we are referring to the time at which the light actually entered the observer's eyes, not the time at which the flashes actually occurred.

Einstein's Train

Now imagine that a (very fast) train is travelling along the track in the direction from A toward B and it so happens that the lightning flashes at A and B hit the ends of the train. The question is: "Do the flashes hit the train simultaneously?" As far as our observer Mike is concerned, as he saw the flashes together the answer must be "yes". If the flashes hit the ends of the train, the ends must have been at A and B at the moments of the flashes. But what of an observer N, Nina, inside the train, let us say at the mid point of the train?

The same definition of simultaneity applies in the train's frame of reference. If the observer sees two flashes which have travelled equal distances at the same time they must have been simultaneous in that frame of reference.

So, do observers in the train also see the two lightning strokes A and B as simultaneous? Imagine that Nina happens to be opposite Mike, that is, also half way between A and B at the moment the flashes occurred (as determined in the embankment frame). See diagram M1. This is NOT the time at which Mike and Nina see the flashes. They see them a little after this moment when the light reaches them – we need to take into account the 'look-back time', that is, the time taken for light to travel from the flashes to the observer.

For Mike to see the events as simultaneous, the light must have come from A and B and met at his position. Remember that Mike is at rest relative to the embankment. Nina in the train, however, is racing away from A and towards B and so will see the flash from B first (diagram M2) because it will have less distance to travel. Note that we could not take a photo and see what is represented in the diagrams! (The camera only 'sees' the light when it enters the lens.) They must be seen as 'reconstructions' of what must have been. Diagram M3 shows the moment that Mike sees both flashes and diagram M4 shows the moment a little later again when Nina sees the flash from A.

Another thing to contemplate is that there really is no need for a train. The train is simply a physical object to help us think about a different reference frame. In reality it could be a car or a spaceship – or simply a hypothetical observer travelling through space.

There is also another small problem – everything we have said so far would be true if A and B were bells that struck just as the ends of the train went by and we were talking about the speed of sound! Mike

M1

M2

M3

M4
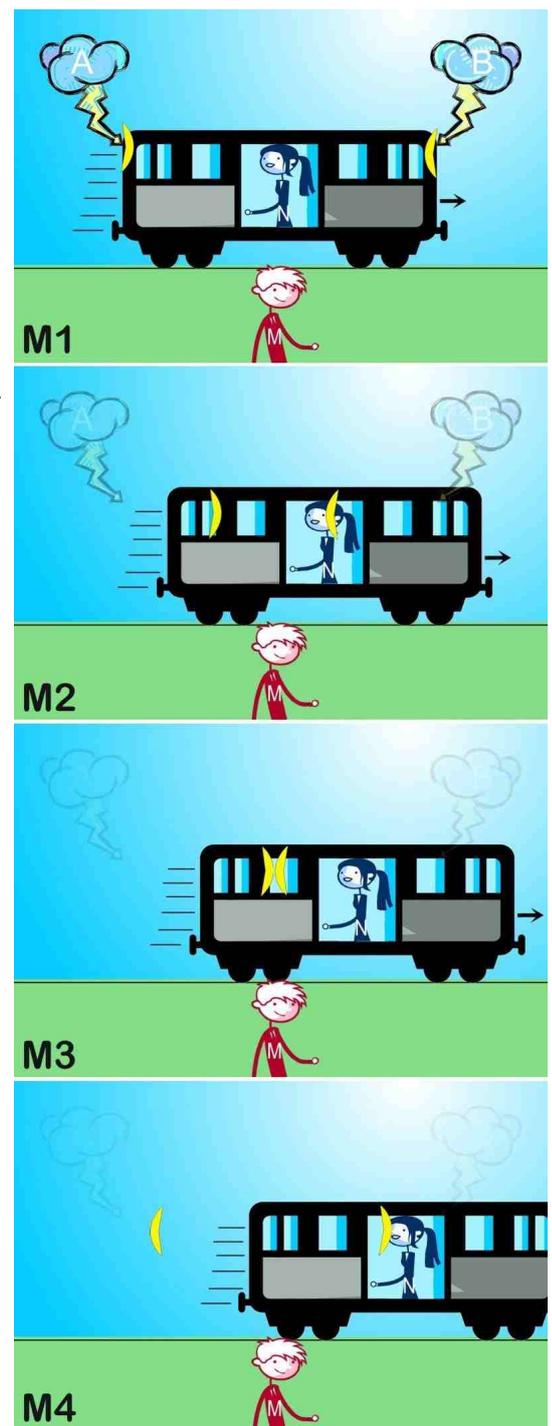
would hear the bells simultaneously and Nina would hear bell B first. However, Nina would realise that she was travelling through the air and so would be able to calculate that in fact the bells struck at the same time.

Likewise, in the case of the light flashes, Nina saw the flash from B first, but will be able to figure out, given that she knows the various distances and speeds involved, that the flashes at A and B would have actually occurred at the same time. So where is the need for Einstein's relativity?

Those Einstein postulates!

Notice that in the previous paragraphs both Mike and Nina assumed that Nina was moving. In other words, they were both looking at the situation from the same frame of reference – Mike's frame, the railway embankment. But what happens if they look at the situation from the frame of reference of the train – Nina's frame? Einstein's first postulate says that all inertial frames of reference are equivalent – there is no 'preferred' frame. So we can equally look at the situation from Nina's point of view, that is, that she is at rest and Mike is whizzing along beside the track.

This is where the difference between the situation with light and with sound enters. If we were talking about bells at A and B, Nina would take into account that the sound travels through air at a fixed speed relative to the air. She would have to add the velocity of the air to that of the sound to obtain the velocity of sound relative to her train.

Relativity makes life simpler! There is **no medium** in which the light is travelling and the speed of light is always 3×10^8 m/s no matter what. That is Einstein's second postulate – the speed of light is the same for all observers.

Let's assume that Mike and Nina are intelligent physicists who are aware of Einstein's two postulates. Nina will still, of course, see the flashes in the same order – flash B before flash A. Likewise, although Mike is now moving (toward A) he must still see the two flashes together. The fact that we are looking from a different frame can not change what the two people actually see. It is their **deductions** about when the flashes occurred that will differ!

In order to deduce when the flashes actually occurred, the two observers can use their knowledge of physics and their measurements of the times and distances involved to calculate the actual times of the flashes. We will not do that, but we can use another set of simple diagrams like that used above. Remember that we are taking into account the look-back times and deducing what must have been happening before the observers actually see the light flashes!

We know that Nina is at rest this time and saw flash B first. Because they occurred at equal distances from her (the ends of the train) she deduces that B must indeed have occurred first (diagram N1). Remember, and this is the big difference between light and other waves, that she knows that light is still travelling at c, the same speed for both flashes, as in the previous example.

A little while later (diagram N2) Nina actually sees the flash from B, and deduces (from what she sees later) that flash A must have occurred. Shortly after that (diagram N3) the flashes were in the same place at the same time and this is when Mike saw them. A little while later again (diagram N4) Nina sees the flash from A.

This is what is remarkable about relativity. What, from one frame of reference (the ground) were deduced to be two simultaneous flashes were not simultaneous in another (the train)! Remember that this is not just a case of who 'sees' what first. Mike would also **deduce** that flash B must have occurred first given that he knows that he is the one in motion. He saw them at the same time at a place closer to A than B and so deduces that B must have occurred earlier as it has had further to travel.

Also remember that this is not simply a case of some difference in 'look-back time', a delay due to the distance light has to travel. It is a fundamental difference in describing a situation from two different frames of reference and is a consequence of the second postulate. The
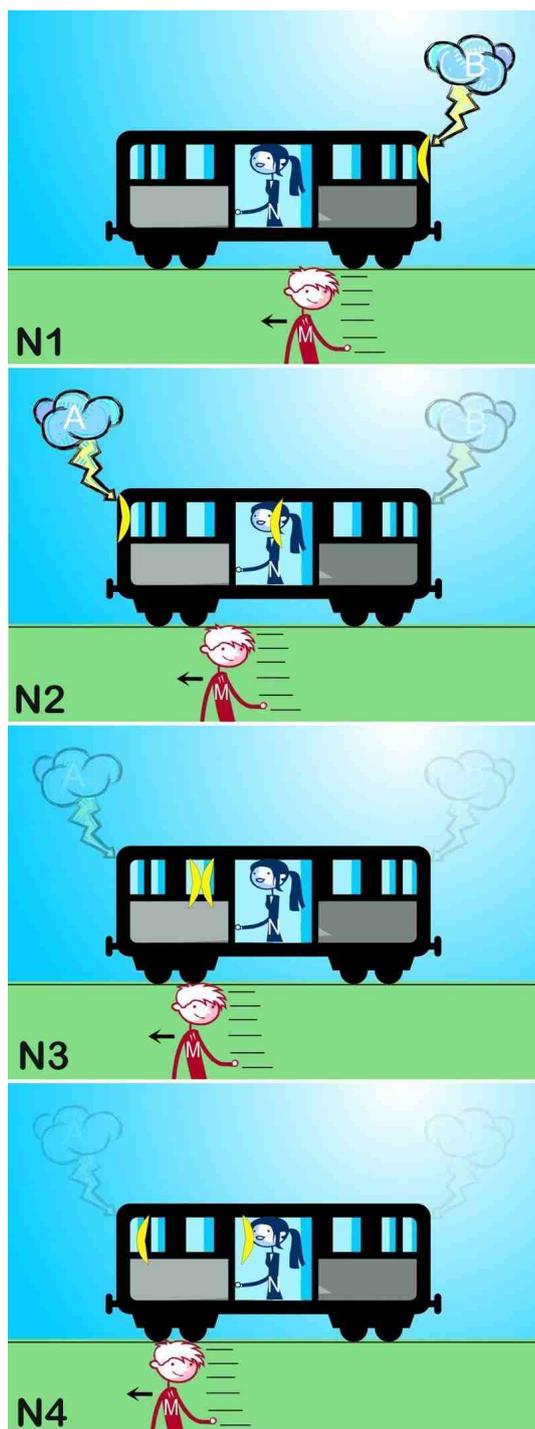
only conclusion can be that it is **time** itself that is acting differently in the two frames.

Whether the observers 'see' the flashes separately or simultaneously depends on their position in the train or along the embankment. If they are indeed in the centre (of their own frame of reference) they will see the flashes in the correct sequence – as Mike does in the first case (simultaneously) and Nina does in the second (B then A). If they were in different positions along the track or the train they may see one or the other flash first, but would deduce that the flashes must have occurred in the same sequence as above.

To repeat, the important conclusion we reach is that what was simultaneous in one frame was not simultaneous in the other! Remember that this is not a case of what various observers saw first, it was a case of what they deduced actually happened. If two events which were simultaneous in one frame were not simultaneous in another what does that mean? The answer must be that time is behaving differently in the two frames. This is the point of these 'Gedanken' experiments – they help us to get our minds around what is really a very weird concept.

Line diagrams

The diagram on the left below is a different way of describing the same situation as we had above (diagrams M1 to M4). The slopes of the various lines indicate the speeds. In this diagram Mike is at rest – we are looking at the situation from his frame. Nina is moving toward B as shown by the slope of her line. The slope of the light lines represents the speed of light, *c*. (Note that the slopes are not 'equal' to the speeds, the diagrams are not the usual x-t graph.) The order of events can be seen from the 'star circles': Nina sees flash B, Mike sees both flashes, Nina sees flash A.

In the right hand diagram we see the situation from Nina's frame of reference. According to her Mike is moving toward A. She knows that the flashes occurred at the ends of the train, that is, at equal distances from her position in the centre of the train. Given that she saw flash B first, she deduces that it must have occurred before flash A (as shown by the fact that it is higher in the diagram). Notice that the order of the events is still the same as in the first diagram.
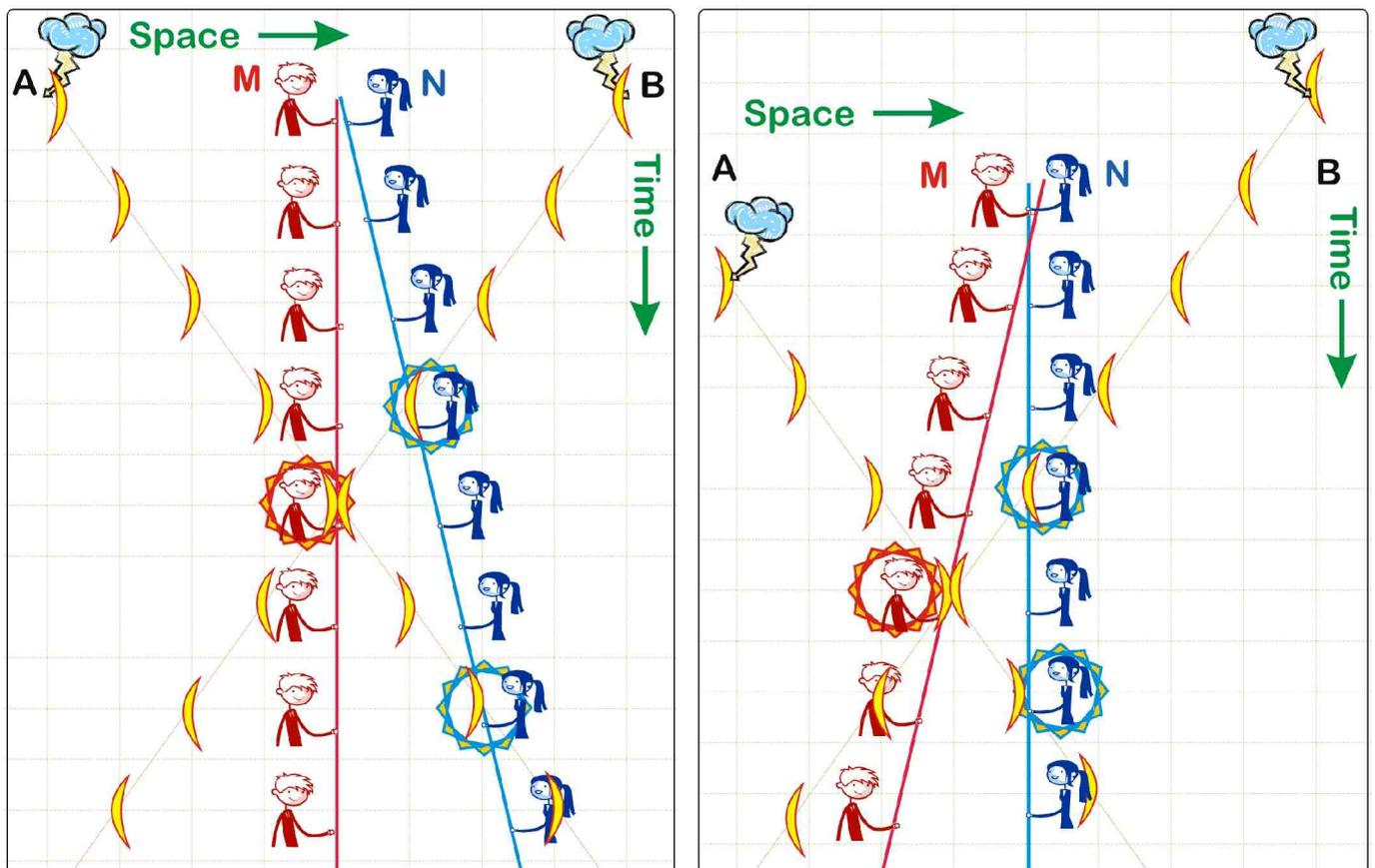

It is worth pointing out that while these types of diagrams can be used to sort out the general sequence of events as seen from different frames of reference, they take no account of length contraction and time dilation and so can not be used in a quantitative sense.

In the situation we have discussed so far, our two observers saw the flashes at different times and places. There were three 'events': Mike seeing both flashes, Nina seeing flash B and Nina seeing flash A. Let us now consider a different scenario. Imagine that instead of the flashes actually occurring at the moment Nina is opposite Mike, the flashes arrive at Mike and Nina at that moment. In other words, Mike saw the flashes at the same moment as Nina passed him. Because Nina passing and the flashes arriving was at the same moment and place these were all one 'event'. As Nina was present at this 'event', she must have seen the flashes at the same moment. If this was the case, what will the two observers say about the simultaneity (or not) of the flashes at A and B this time?

As far as observer Mike is concerned this case is similar to the last. He is at rest relative to the embankment and half way between A and B. He must conclude that the flashes were simultaneous. But what of observer Nina in the train?

The situation for Mike and Nina is shown in the two diagrams below. The left diagram shows us Mike's point of view and the right, the situation from Nina's frame of reference.
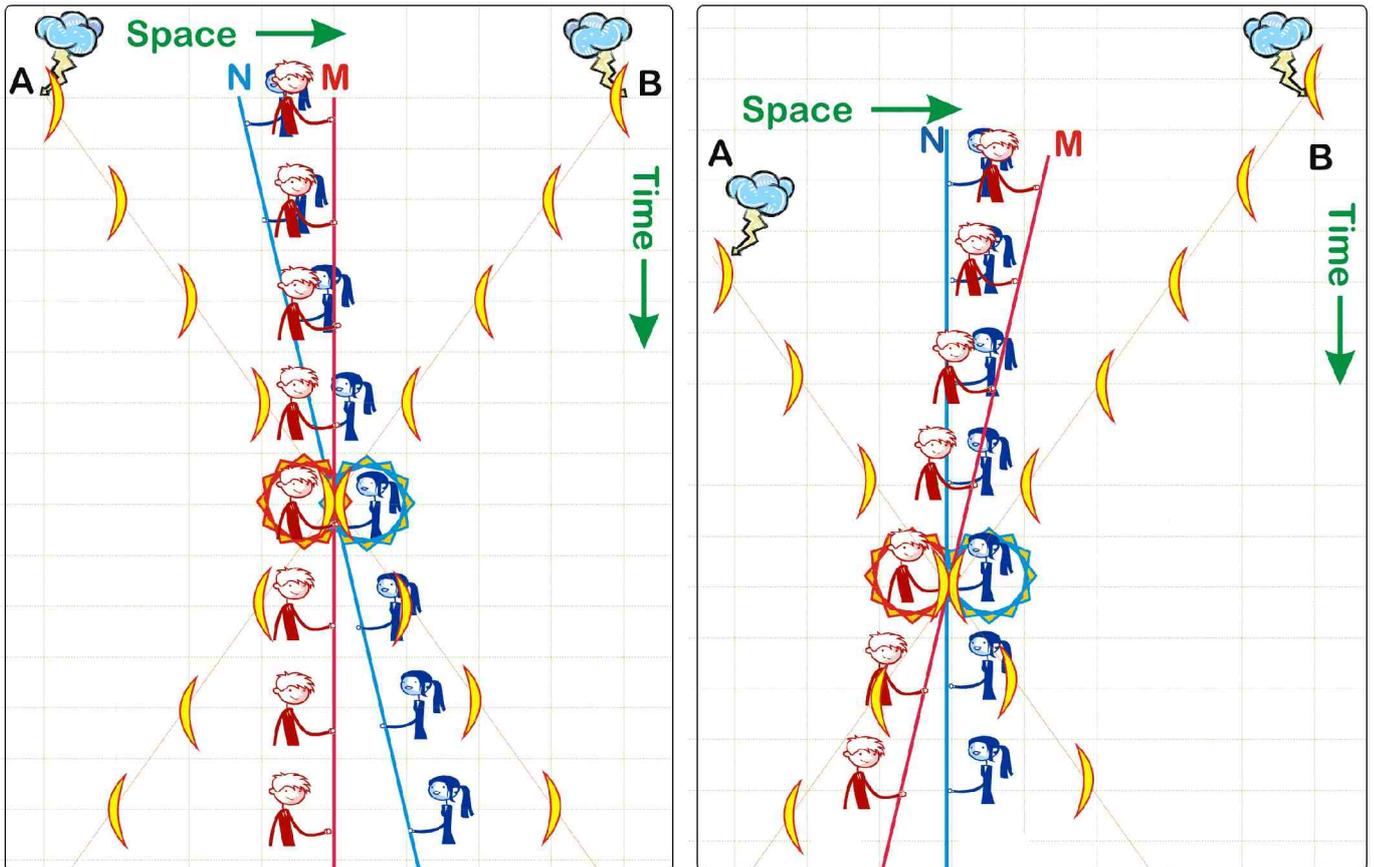
In fact, we find that the situation is much the same as before. As Mike is still mid-way between A and B he says the flashes were simultaneous. Nina will agree with Mike (when they talk about it later) as far as his frame is concerned, but will say that in her frame flash B must have occurred first. There is a simple reason for this. The only difference between this and the previous case is that Nina was further toward A when the actual flashes occurred – as shown by the top line in the diagrams above. Compare the two diagrams on this page with those on the previous page and you will see that the only real difference is that Nina started further to the left (from M's point of view) or Mike started further to the right (from N's point of view). This could have either been because N was not in the centre of the train, or because the ends of the train were not at A and B when the lightning flashes occurred.

So far, our trains have experienced mysterious simultaneous (or not) lightning flashes at either end. You may think that's a little far fetched. Remember, however, that these are 'gedanken' experiments - we can think as far as we like (no fetching involved). The more prosaic amongst us might prefer the next example – we have a simple light bulb in the middle of the carriage that can be switched on whenever we like. Mind you, our train is still going rather fast!

A different train

A different version of the train Gedanken has the flashing light in the centre. Just as the train passes an external observer (C) the light flashes. As can be seen in these diagrams (right) the light from the flash takes a little while to reach the observers at either end of the train. However, because of the motion of the train, the light reaches A (at the left hand end) before it reaches B. Remember that observer C will have to take into account the look-back time for the light to reach her from the points where it reflects off A and B, but she will deduce that the light was seen by A first.

So in the frame of a stationary observer outside the train, A will see the flash before B does. But what about the observers inside the train? What do they say from their frame of reference?

This is a very simple situation. The observers in the train are equal distances from the light and so must see the flash at the same time (diagrams below). The speed of light, as always, is the same for both of them and the distances are equal.

A and B see observer C speeding by in the other direction. If they were to calculate how observer C saw the flash they would be able to work out that she must have been opposite the light when it flashed and so would have seen a single flash as she passed the light – just as observer C would have said.

Notice that while there was only the one event of C seeing the flash, there were two different events for A and B seeing the flashes (light arriving at A and light arriving at B). C saw the flash as she passed the light. The seeing and the passing occurred at the same time and same place (same event) and so there can be no different versions of that. However, the two different events of A and B seeing the flashes are separated in space and time and so, as we have seen, may be perceived differently by different observers. Indeed while any observers inside the train will conclude that the flashes reached the ends of the train at the same time (in the train frame) any observers outside will decide that the flash reached the left hand end first (in the ground frame). Again, the only explanation is that time itself behaves differently in different frames of reference.
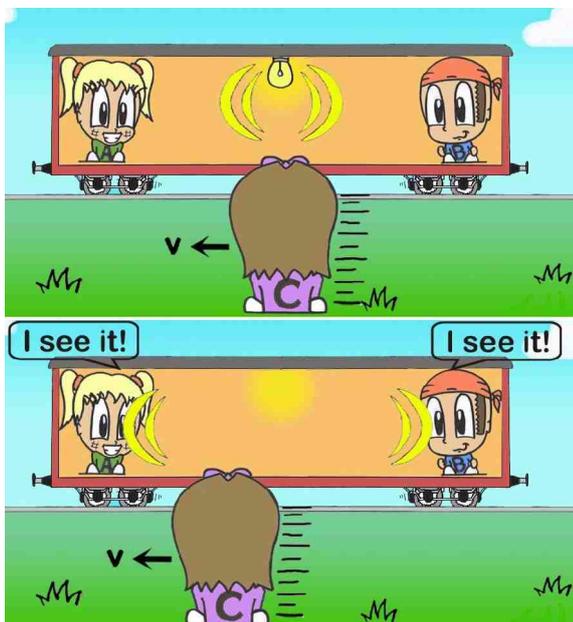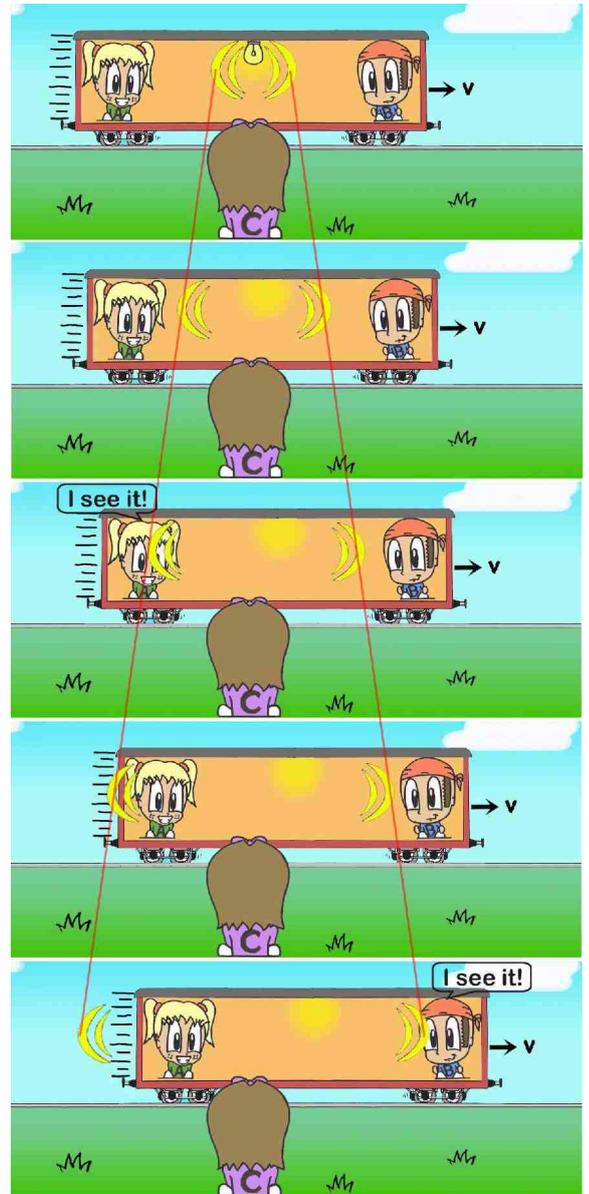
It was by analysing situations like these that Einstein discovered his famous time dilation and length contraction equations. It is worth noting that, while it makes no difference to our qualitative descriptions, all the moving trains we have been discussing should be 'length contracted' due to their speed. Indeed, we can use further 'thought experiments' to see that the length of a fast moving train (or any object) must be contracted in the direction of its motion.